

**PART-III**  
**MATHEMATICS**  
**PAPER-I (A)**

**MODEL**  
**PAPER | 1**

Time : 3 Hours

Max. Marks : 75

**Note :** This question paper consists of **three** sections **A, B** and **C**.

**SECTION - A** (10 × 2 = 20)

I. **Very short** answer type questions.

(i) Attempt **all** questions.

(ii) Each question carries **two** marks.

1. If  $A = \{-2, -1, 0, 1, 2\}$  and  $f: A \rightarrow B$  is a surjection defined by  $f(x) = x^2 + x + 1$ , then find  $B$ .
2. If  $A = \left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right\}$  and  $f: A \rightarrow B$  is a surjection defined by  $f(x) = \cos x$  then find  $B$ .
3. If  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ ,  $B = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$  and  $2X + A = B$  then find  $X$ .
4. If  $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & -2 \\ -1 & x & 0 \end{bmatrix}$  is a skew symmetric matrix, then find  $x$ .
5. If the vectors  $-3i + 4j + \lambda k$  and  $\mu i + 8j + 6k$  are collinear vectors, then find  $\lambda$  and  $\mu$ .
6. Find the vector equation of the line passing through the point  $2i + 3j + k$  and parallel to the vector  $4i - 2j + 3k$ .
7. Find the angle between the vectors  $i + 2j + 3k$  and  $3i - j + 2k$ .
8. If  $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$ , then prove that  $\cos \theta - \sin \theta = \sqrt{2} \sin \theta$ .
9. If  $A + B = \frac{\pi}{4}$ , then prove that  $(1 + \tan A)(1 + \tan B) = 2$ .
10. If  $\cosh x = \frac{5}{2}$ , find the values of  
(i)  $\cosh(2x)$  and (ii)  $\sinh(2x)$

**SECTION - B** (5 × 4 = 20)

II. **Short answer** type questions.

(i) Attempt any **five** questions.

(ii) Each question carries **four** marks.

11. If  $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$  then show that  $A^2 - 4A - 5I = 0$

12. If  $a, b, c$  are non-coplanar vectors. Prove that the following four points are coplanar.

**Intermediate First Year (Mathematics-1(A))**

13. Find the area of the triangle whose vertices are A(1, 2, 3), B(2, 3, 1) and C(3, 1, 2).
14. If A is not an integral multiple of  $\pi$ . Prove that

$$\cos A \cdot \cos 2A \cdot \cos 4A \cdot \cos 8A = \frac{\sin 16A}{16 \sin A} \text{ and hence deduce that}$$

$$\cos \frac{2\pi}{15} \cdot \cos \frac{4\pi}{15} \cdot \cos \frac{8\pi}{15} \cdot \cos \frac{16\pi}{15} = \frac{1}{16}$$

15. Solve  $\sqrt{2}(\sin x + \cos x) = \sqrt{3}$
16. Prove that  $2 \sin^{-1} \frac{3}{5} - \cos^{-1} \frac{5}{13} = \cos^{-1} \frac{323}{325}$
17. If  $\sin \theta = \frac{a}{b+c}$ , then show that  $\cos \theta = \frac{2\sqrt{bc}}{b+c} \cos \frac{A}{2}$ .

**SECTION - C (5 × 7 = 35)**

III. Long answer type questions.

- (i) Attempt any five questions.
- (ii) Each question carries seven marks.

18. If  $f: A \rightarrow B$ ,  $g: B \rightarrow C$  are bijections then prove that  $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ .

19.  $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \dots$  upto n terms =  $\frac{n(n+1)^2(n+2)}{12}$

20. Show that  $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$ .

21. Solve  $3x + 4y + 5z = 18$ ,  $2x - y + 8z = 13$  and  $5x - 2y + 7z = 20$  by using matrix inversion method.
22. Find equation of plane passing from points A (2, 3, -1), B (4, 5, 2) and C (3, 6, 5)
23. If  $A + B + C = 180^\circ$  then show that  $\cos 2A + \cos 2B + \cos 2C = -4 \cos A \cos B \cos C - 1$ .
24. If  $a = 13$ ,  $b = 14$ ,  $c = 15$ , show that  $R = \frac{65}{8}$ ,  $r = 4$ ,  $r_1 = \frac{21}{2}$ ,  $r_2 = 12$  and  $r_3 = 14$ .

PART-III  
MATHEMATICS  
PAPER-I (A)

MODEL  
PAPER 2

Time : 3 Hours

Max. Marks : 7

**Note :** This question paper consists of three sections A, B and C.

**SECTION - A** (10 × 2 = 20)

I. Very short answer type questions.

- (i) Attempt all the questions.  
(ii) Each question carries two marks.

1. If  $f:R \rightarrow R$ ,  $g:R \rightarrow R$  are defined by  $f(x) = 3x - 1$ ,  $g(x) = x^2 + 1$ , then find  $f \circ g(2)$ .
2. Find the domains of the following real valued functions  $f(x) = \log(x - 4x + 3)$
3. Construct a  $3 \times 2$  matrix, whose elements are defined by  $a_{ij} = \frac{1}{2} |i - 3j|$
4. Find the rank of  $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$
5.  $a = 2i + 5j + k$  and  $b = 4i + mj + nk$  are collinear vectors, then find  $m$  and  $n$ .
6. Find the vector equation of the line joining the point  $2i + j + 3k$  and  $-4i + 3j - k$ .
7. Find the area of the parallelogram for which the vectors  $\vec{a} = 2\vec{i} - 3\vec{j}$  and  $\vec{b} = 3\vec{i} - \vec{k}$  are adjacent sides.
8. Find the period of the function  $\tan(x + 4x + 9x + \dots + n^2x)$  [  $n$  any positive integer]
9. Prove that  $\frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = \cot 36^\circ$ .
10. If  $\sinh x = \frac{3}{4}$ , find  $\cosh(2x)$  and  $\sinh(2x)$ .

**SECTION - B** (5 × 4 = 20)

II. Short answer type questions.

- (i) Attempt any five questions.  
(ii) Each question carries four marks.

11. If  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  and  $E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$  then show that  $(aI + bE)^3 = a^3I + 3a^2bE$ , where  $I$  is unit matrix of order 2.
12. If ABCDEF is a regular hexagon with centre 'O' then show that  $AB + AC + AD + AE + AF = 3AD = 6AO$ .

13. Find the volume of the tetrahedron having the edges  $i + j + k$ ,  $i - j$  and  $i + 2j + k$ .
14. Prove that  $\left(1 + \cos \frac{\pi}{10}\right) \left(1 + \cos \frac{3\pi}{10}\right) \left(1 + \cos \frac{7\pi}{10}\right) \left(1 + \cos \frac{9\pi}{10}\right) = \frac{1}{16}$
15. If  $\theta_1, \theta_2$  are solutions of the equation  $a \cos 2\theta + b \sin 2\theta = c$ ,  $\tan \theta_1 \neq \tan \theta_2$  and  $a + c \neq 0$ , then find the values of
- $\tan \theta_1 + \tan \theta_2$
  - $\tan \theta_1 \cdot \tan \theta_2$
16. Prove that  $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$
17. Prove that  $\cot A + \cot B + \cot C = \frac{a^2 + b^2 + c^2}{4\Delta}$

**SECTION - C** ( $5 \times 7 = 35$ )

III. Long answer type questions.

- Attempt any five questions.
- Each question carries seven marks.

---

18. If  $f : A \rightarrow B$ ,  $g : B \rightarrow C$  are bijections then prove that  $g \circ f : A \rightarrow C$  is a bijection
19. Show that,  $\forall n \in \mathbb{N}$ ,  $\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots$  upto  $n$  terms  $= \frac{n}{3n+1}$ .
20. Show that  $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$  is non singular and find  $A^{-1}$ .
21. Solve  $2x - y + 3z = 9$   
 $x + y + z = 6$   
 $x - y + z = 2$  by using Cramer's rule
22. Find the shortest distance between the skew lines  
 $r = (6i + 2j + 2k) + t(i - 2j + 2k)$  and  $r = (-4i - k) + s(3i - 2j - 2k)$ .
23. If  $A + B + C = \pi$ , then prove that  $\cos^2 \frac{A}{2} + \cos^2 \frac{B}{2} + \cos^2 \frac{C}{2} = 2 \left(1 + \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}\right)$
24. If  $r_1 = 2$ ,  $r_2 = 3$ ,  $r_3 = 6$  and  $r = 1$ , Prove that  $a = 3$ ,  $b = 4$  and  $c = 5$ .

**PART-III**  
**MATHEMATICS**  
**PAPER-I (A)**

**MODEL**  
**PAPER** | **3**

Time : 3 Hours

Max. Marks : 75

**Note :** This question paper consists of **three** sections **A, B** and **C**.

**SECTION - A** (10 × 2 = 20)

I. **Very short** answer type questions.

(i) Attempt **all** the questions.

(ii) Each question carries **two** marks.

- 
1. Which of the following are injections or surjections or bijections? Justify your answers.  
 $f: \mathbb{R} \rightarrow (0, \infty)$  defined by  $f(x) = 2^x$
  2. Let  $f = \{(1, a), (2, c), (4, d), (3, b)\}$  and  $g^{-1} = \{(2, a), (4, b), (1, c), (3, d)\}$ , then show that  $(gof)^{-1} = f^{-1}og^{-1}$
  3. If  $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ , then find  $A^4$ .
  4. For any square matrix  $A$ , show that  $AA'$  is symmetric.
  5. If  $\alpha, \beta$  and  $\gamma$  are the angles made by the vector  $3i - 6j + 2k$  with the positive directions of the coordinate axes then find  $\cos \alpha, \cos \beta$  and  $\cos \gamma$ .
  6. If the position vectors of the points  $A, B$  and  $C$  are  $-2i + j - k, -4i + 2j + 2k$  and  $6i - 3j - 13k$  respectively and  $AB = \lambda AC$ , then find the value of  $\lambda$ .
  7. If the vectors  $2i + \lambda j - k$  and  $4i - 2j + 2k$  are perpendicular to each other, find  $\lambda$ .
  8. Find a sine function whose period is  $\frac{2\pi}{3}$ .
  9. If  $\tan 20^\circ = \lambda$ , then show that  $\frac{\tan 160^\circ - \tan 110^\circ}{1 + \tan 160^\circ \tan 110^\circ} = \frac{1 - \lambda^2}{2\lambda}$
  10. If  $\sinh x = 3$ , then show that  $x = \log_e (3 + \sqrt{10})$

**SECTION - B** (5 × 4 = 20)

II. **Short answer** type questions.

(i) Attempt any **five** questions.

(ii) Each question carries **four** marks.

- 
11. If  $\theta - \phi = \frac{\pi}{2}$  then show that  $\begin{bmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{bmatrix} \begin{bmatrix} \cos^2 \phi & \cos \phi \sin \phi \\ \cos \phi \sin \phi & \sin^2 \phi \end{bmatrix} = 0$ .
  12. Show that the points whose position vectors are  $-2a + 3b + 5c, a + 2b + 3c, 7a - c$  are collinear when  $\vec{a}, \vec{b}, \vec{c}$  are non coplanar vectors.

**Intermediate First Year (Mathematics-I(A))**

13. If  $a, b, c$  are non coplanar vectors then prove that the vectors  $5a + 6b + 7c, 7a - 8b + 9c$  and  $3a + 20b + 10c$  are coplanar
14.  $0 < A < B < \frac{\pi}{4}$  and  $\sin(A + B) = \frac{24}{25}$  and  $\cos(A - B) = \frac{4}{5}$ , then find the value of  $\tan 2A$ .
15. Solve  $1 + \sin^2\theta = 3\sin\theta \cos\theta$ .
16. If  $\cos^{-1}\left(\frac{p}{a}\right) + \cos^{-1}\left(\frac{q}{b}\right) = \alpha$  then prove that  $\frac{p^2}{a^2} - \frac{2pq}{ab} \cdot \cos\alpha + \frac{q^2}{b^2} = \sin^2\alpha$ .
17. If  $\cot\frac{A}{2} : \cot\frac{B}{2} : \cot\frac{C}{2} = 3 : 5 : 7$  then show that  $a : b : c = 6 : 5 : 4$  (In  $\Delta ABC$ ).

**SECTION - C (5 × 7 = 35)**

III. Long answer type questions.

(i) Attempt any five questions.

(ii) Each question carries seven marks.

18. Let  $f : A \rightarrow B, I_A$  and  $I_B$  be identity functions on  $A$  and  $B$  respectively. Then prove that  $f \circ I_A = f = I_B \circ f$
19.  $3 \cdot 5^{2n+1} + 2^{3n+1}$  is divisible by 17
20. Solve the following equations by Gauss Jordan method.

$$3x + 4y + 5z = 18, 2x - y + 8z = 13, 5x - 2y + 7z = 20$$

21. Show that 
$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = (a^3 + b^3 + c^3 - 3abc)^2$$

22. If  $A = (1, -1, 1), B = (4, 0, -3), C = (1, 2, -1)$  and  $D = (2, -4, -5)$ , find the distance between  $AB$  and  $CD$

23. In triangle  $ABC$ , prove that

$$\cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2} = 4\cos\frac{\pi-A}{4} \cos\frac{\pi-B}{4} \cos\frac{\pi-C}{4}$$

24. If  $p_1, p_2, p_3$  are altitudes drawn from vertices  $A, B, C$  to the opposite sides of a triangle respectively, the show that

(i)  $\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3} = \frac{1}{r}$  (ii)  $\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3} = \frac{1}{r}$  (iii)  $p_1 p_2 p_3 = \frac{(abc)^2}{8R^3} = \frac{8\Delta^3}{abc}$

**PART-III**  
**MATHEMATICS**  
**PAPER-I(A)**

**GUESS**  
**PAPER** | **1**

Time : 3 Hours

Max. Marks : 75

**SECTION - A** (10 × 2 = 20)

- Note :**
- (i) Answer all questions.
  - (ii) Each question carries two marks.
  - (iii) All are Very Short Answer Type Questions.

---

1. If  $f : \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = \frac{1-x^2}{1+x^2}$ , then show that  $f(\tan \theta) = \cos 2\theta$
2. If  $f(x) = \frac{1}{x}$ ,  $g(x) = \sqrt{x}$  for all  $x \in (0, \infty)$ , then find  $(g \circ f)(x)$ .
3. If  $\begin{bmatrix} x-1 & 2 & 5-y \\ 0 & z-1 & 7 \\ 1 & 0 & a-5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 1 & 0 & 0 \end{bmatrix}$  then find the values of  $x$ ,  $y$ ,  $z$  and  $a$ .
4. If  $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$ , find  $A^2$ .
5. Is the triangle formed by the vectors  $3i + 5j + 2k$ ,  $2i - j - k$  and  $-5i - 2j + 3k$  equilateral ?
6. Define linear combination of vectors.
7. Find the angle between the planes  $r \cdot (2i - j + 2k) = 3$  and  $r \cdot (3i + 6j + k) = 4$ .
8. Find the value of  $\sin 330^\circ \cdot \cos 120^\circ + \cos 210^\circ \cdot \sin 300^\circ$
9. If  $\sec \theta + \tan \theta = 5$ , find the quadrant in which  $\theta$  lies and find the value of  $\sin \theta$ .
10. Show that  $\tanh^{-1} \left( \frac{1}{2} \right) = \frac{1}{2} \ln \frac{3}{5}$ .

**SECTION - B** (5 × 4 = 20)

- Note :** Short answer type questions.
- (i) Attempt any five questions.
  - (ii) Each question carries four marks.

---

11. If  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  and  $E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$  then show that  $(aI + bE)^3 = a^3I + 3a^2bE$ , where  $I$  is unit matrix of order 2.
12. If the points whose position vectors are  $3i - 2j - k$ ,  $2i + 3j - 4k$ ,  $-i + j + 2k$  and  $4i + 5j + \lambda k$  are coplanar then show that  $\lambda = \frac{-146}{17}$ .

### Intermediate First Year (Mathematics-1(A))

13. Prove by vector method, the angle between the two diagonals of a cube  $\cos^{-1}\left(\frac{1}{3}\right)$
14. If  $\cos \alpha = \frac{-3}{5}$  and  $\sin \beta = \frac{7}{25}$ , where  $\frac{\pi}{2} < \alpha < \pi$  and  $0 < \beta < \frac{\pi}{2}$ , then find the values of  $\tan(\alpha + \beta)$  and  $\sin(\alpha + \beta)$ .
15. Solve  $4\sin x \cdot \sin 2x \cdot \sin 4x = \sin 3x$ .
16. Prove that  $\tan \left\{ \cot^{-1} 9 + \operatorname{cosec}^{-1} \frac{\sqrt{41}}{4} \right\} = 1$
17. If  $a : b : c = 7 : 8 : 9$ , find  $\cos A : \cos B : \cos C$ .

### SECTION - C (5 × 7 = 35)

**Note :** Long answer type questions.

(i) Attempt any **five** questions.

(ii) Each question carries **seven** marks.

---

18. If  $f: \mathbb{Q} \rightarrow \mathbb{Q}$  is defined by  $f(x) = 5x + 4$ ,  $\forall x \in \mathbb{Q}$ , show that  $f$  is a bijection and find  $f^{-1}$ .
19.  $2.3 + 3.4 + 4.5 + \dots$  upto  $n$  terms  $= \frac{n(n^2 + 6n + 11)}{3}$
20. Show that  $\begin{vmatrix} -2a & a+b & c+a \\ a+b & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix} = 4(a+b)(b+c)(c+a)$
21.  $x - 3y - 8z = -10$   
 $3x + y - 4z = 0$   
 $2x + 5y + 6z = 13$
22. If  $a = i - 2j + k$ ,  $b = 2i + j + k$ ,  $c = i + 2j - k$ , find  $a \times (b \times c)$  and  $|(a \times b) \times c|$ .
23. In triangle ABC, prove that  $\sin \frac{A}{2} + \sin \frac{B}{2} - \sin \frac{C}{2} = -1 + 4 \cos \frac{\pi - A}{4} \cos \frac{\pi - B}{4} \sin \frac{\pi - C}{4} \omega$
24. If  $a^2 + b^2 + c^2 = 8R^2$ , then prove that the triangle is right angled.



**PART-III**  
**MATHEAMTICS**  
**PAPER-I(A)**

GUESS  
PAPER | 2

Time : 3 Hours

Max. Marks : 75

**SECTION - A** (10 × 2 = 20)

- Note :**
- (i) Answer **all** questions.
  - (ii) Each question carries **two** marks.
  - (iii) All are **Very Short Answer Type Questions**.

1. Find the domains of the following real valued functions.

$$f(x) = \frac{1}{\log(2-x)}$$

2. If  $A = \{1, 2, 3, 4\}$  and  $f: A \rightarrow \mathbb{R}$  is a function defined by  $f(x) = \frac{x^2 - x + 1}{x + 1}$ , then find the range of  $f$ .

3. If  $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ , then find  $A^3$

4. If  $A = \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}$  and  $A^2 = 0$ , then find the value of  $k$ .

5. Find the unit vector in the direction of vector

$$\vec{a} = 2\mathbf{i} + 3\mathbf{j} + k$$

6. Find the vector equation of the line passing through the point  $2\mathbf{i} + 3\mathbf{j} + k$  and parallel to the vector  $4\mathbf{i} - 2\mathbf{j} + 3k$ .

7. Let  $e_1$  and  $e_2$  be unit vectors making an angle  $\theta$ . If  $\frac{1}{2}|e_1 - e_2| = \sin \lambda\theta$ , then find  $\lambda$ .

8. Prove that

$$\frac{\sin 150^\circ - 5\cos 300^\circ + 7\tan 225^\circ}{\tan 135^\circ + 3\sin 210^\circ} = \frac{1}{2}$$

9. Prove that

$$(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 - (\tan^2\theta + \cot^2\theta) = 7$$

10. If  $\sin hx = 5$ , then show that  $x = \log_e(5 + \sqrt{26})$ .

**SECTION - B** (5 × 4 = 20)

**Note :** Short answer type questions.

- (i) Attempt any **five** questions.
- (ii) Each question carries **four** marks.

11. If  $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$  then show that  $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$  for any integer  $n \geq 1$ , by using mathematical induction.

Intermediate First Year (Mathematics-1(A))

12. Find the vector equation of the plane passing through points  $4i - 3j - k$ ,  $3i + 7j - 10k$  and  $2i + 5j - 7k$  and show that the point  $i + 2j - 3k$  lies in the plane.
13. If  $a = 2i + j - k$ ,  $b = -i + 2j - 4k$  and  $c = i + j + k$ , then find  $(a \times b) \cdot (b \times c)$
14. Evaluate  
 $\sin^2 82 \frac{1}{2}^\circ - \sin^2 22 \frac{1}{2}^\circ$
15. Solve the following equation  $\cot x + \operatorname{cosec} x = \sqrt{3}$
16. Prove that  $\tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{13} - \tan^{-1} \frac{2}{9} = 0$
17. Show that  $a \cos^2 \frac{A}{2} + b \cos^2 \frac{B}{2} + c \cos^2 \frac{C}{2} = s + \frac{\Delta}{R}$ .

**SECTION - C** (5 × 7 = 35)

Note : Long answer type questions.

(i) Attempt any five questions.

(ii) Each question carries seven marks.

18. Let  $f : A \rightarrow B$  be a bijection, then show that  $f \circ f^{-1} = I_B$  and  $f^{-1} \circ f = I_A$ .
19. By Mathematical induction, show that  $49^n + 16n - 1$  is divisible by 64 for all positive integer  $n$ .
20. Solve the following equations, by Cramer's Rule  

$$\begin{aligned} x + y + z &= 1 \\ 2x + 2y + 3z &= 6 \\ x + 4y + 9z &= 3 \end{aligned}$$
21. Examine whether the or following systems of equations are consistent or inconsistent and if consistent find the complete solution.  

$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 10 \\ x + 2y + 4z &= 1 \end{aligned}$$
22. If  $A = (1, -2, -1)$ ,  $B = (4, 0, -3)$ ,  $C = (1, 2, -1)$  and  $D = (2, -4, -5)$ , find the distance between AB and CD.
23. If  $A + B + C = 0$ , then prove that  $\sin 2A + \sin 2B + \sin 2C = -4 \sin A \sin B \sin C$ .
24. Show that  $r + r_3 + r_1 - r_2 = 4R \cos B$  in a triangle ABC.

# BOARD MODEL PAPER

## JR. MATHEMATICS- IA

Time : 3 Hours.

Max. Marks : 75

### SECTION-A

#### I. Very Short Answer Questions.

10 × 2 = 20

1. If  $A = \left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right\}$  and  $f: A \rightarrow B$  is a surjection defined by  $f(x) = \cos x$  then find B.
2. Find the domain of the real-valued function  $f(x) = \frac{1}{\log(2-x)}$ .
3. A certain bookshop has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs. 80, Rs. 60 and Rs. 40 each respectively. Find the total amount the bookshop will receive by selling all the books, using matrix algebra.
4. If  $A = \begin{pmatrix} 2 & -4 \\ -5 & 3 \end{pmatrix}$ , then find  $A+A'$  and  $AA'$ .
5. Show that the point whose position vectors are  $-2\bar{a} + 3\bar{b} + 5\bar{c}$ ,  $\bar{a} + 2\bar{b} + 3\bar{c}$ ,  $7\bar{a} - \bar{c}$  are collinear when  $\bar{a}, \bar{b}, \bar{c}$  are non-coplanar vectors.
6. Let  $\bar{a} = 2\bar{i} + 4\bar{j} - 5\bar{k}$ ,  $\bar{b} = \bar{i} + \bar{k}$  and  $\bar{c} = \bar{j} + 2\bar{k}$ . Find unit vector in the opposite direction of  $\bar{a} + \bar{b} + \bar{c}$ .
7. If  $\bar{a} = \bar{i} + 2\bar{j} - 3\bar{k}$  and  $\bar{b} = -2\bar{j} + 2\bar{k}$  then show that  $\bar{a} + \bar{b}$  and  $\bar{a} - \bar{b}$  are perpendicular to each other.
8. Prove that  $\frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = \cot 36^\circ$ .
9. Find the period of the function defined by  $f(x) = \tan(x + 4x + 9x + \dots + n^2x)$ .
10. If  $\sinh x = 3$  then show that  $x = \log_e(3 + \sqrt{10})$ .

### SECTION-B

#### II. Short Answer Questions.

5 × 4 = 20

11. Show that 
$$\begin{vmatrix} bc & b+c & 1 \\ ca & c+a & 1 \\ ab & a+b & 1 \end{vmatrix} = (a-b)(b-c)(c-a)$$

12. Let ABCDEF be regular hexagon with centre 'O'. Show that

$$\overline{AB} + \overline{AC} + \overline{AD} + \overline{AE} + \overline{AF} = 3\overline{AD} = 6\overline{AO}$$

13. If  $\vec{a} = \vec{i} - 2\vec{j} - 3\vec{k}$ ,  $\vec{b} = 2\vec{i} + \vec{j} - \vec{k}$  and  $\vec{c} = \vec{i} + 3\vec{j} - 2\vec{k}$ , find  $\vec{a} \times (\vec{b} \times \vec{c})$

14. If A is not an integral multiple of  $\frac{\pi}{2}$ , prove that

$$(i) \tan A + \cot A = 2 \operatorname{cosec} 2A \quad (ii) \cot A - \tan A = 2 \cot 2A$$

15. Solve  $2 \cos^2 \theta - \sqrt{3} \sin \theta + 1 = 0$

16. Prove that  $\cos\left(2 \tan^{-1} \frac{1}{7}\right) = \sin\left(4 \tan^{-1} \frac{1}{3}\right)$

17. In a triangle ABC, prove that  $\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c} \cot \frac{A}{2}$

### SECTION C

III. Answer any five of the following:

5 × 7 = 35

18. Let  $f: A \rightarrow B$ ,  $g: B \rightarrow C$  be bijections. Then prove that  $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

19. By using mathematical induction show that  $\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots$  (upto n terms)  $= \frac{n}{3n+1}$ ,  $\forall n \in \mathbb{N}$

20. If  $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & -1 \end{bmatrix}$  then find  $(A^{-1})^{-1}$

21. Solve the following equations by Gauss - Jordan method

$$3x + 4y + 5z = 18, 2x - y + 8z = 13 \text{ and } 5x - 2y + 7z = 20$$

22. If  $A = (1, -2, -1)$ ,  $B = (4, 0, -3)$ ,  $C = (1, 2, -1)$  and  $D = (2, -4, -5)$ , find the distance between  $\overline{AB}$  and  $\overline{CD}$

23. If A, B, C are angles of a triangle, then prove that  $\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} - \sin^2 \frac{C}{2} = 1 - 2 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$

24. In a triangle ABC, if  $a = 13$ ,  $b = 14$ ,  $c = 15$ , find  $R$ ,  $r$ ,  $r_1$ ,  $r_2$  and  $r_3$ .