1. If $A=\left\{0,\frac{\pi}{6},\frac{\pi}{4},\frac{\pi}{3},\frac{\pi}{2}\right\}$ and $f:A\to B$ is a surjection defined by $f\left(x\right)=\cos x$ then find B

sol. Given $f(x)=\cos x$, $f(0)=\cos(0)=1$,

$$f\left(\frac{\pi}{6}\right)=\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2},\ f\left(\frac{\pi}{4}\right)=\cos\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}},\ f\left(\frac{\pi}{3}\right)=\cos\left(\frac{\pi}{3}\right)=\frac{1}{2},\ f\left(\frac{\pi}{2}\right)=\cos\left(\frac{\pi}{2}\right)=0$$

$\therefore$ Range of $B\left\{1,\frac{\sqrt{3}}{2},\frac{1}{\sqrt{2}},\frac{1}{2},0\right\}$

2. If $f:R\to R, g:R\to R$ are defined by $f\left(x\right)=3x-1, g(x)=x^2+1$ then find i)fog(2) ii)gof(2a-3)

sol. (i) Given $g(x)=x^2+1$

$\Rightarrow g(2)=2^2+1$

$\Rightarrow g(2)=5$

$\Rightarrow f(g(2))=f(5)$

$\Rightarrow f(g(2))=3(5)-1 \quad [\because f(x)=3x-1]$

$\Rightarrow fog(2)=15-1=14$

(ii) Given $f(x)=3x-1$

$\Rightarrow f(2a-3)=3(2a-3)-1$

$\Rightarrow f(2a-3)=6a-9-1$

$\Rightarrow f(2a-3)=6a-10$

$\Rightarrow g(f(2a-3))=g(6a-10)$

$\Rightarrow g(f(2a-3))=(6a-10)^2+1 \quad [\because g(x)=x^2+1]$

$\Rightarrow gof(2a-3))=(36a^2-120a+101)$

3. If $f(x)=2, g(x)=x^2, h(x)=2x$ for all $x\in R$, then find $(fo(goh))\{x\}$

sol. Given $f(x)=2, g(x)=x^2, h(x)=2x$

$(fo(goh))\{x\}=f\{g[h(x)]\}=f\{g[2x]\}=f\{4x^2\}=2$

4. Find the domain of the real valued function $f\left(x\right)=\frac{1}{(x^2-1)(x+3)}$

sol. Given $f\left(x\right)=\frac{1}{(x^2-1)(x+3)}=\frac{1}{(x+1)(x-1)(x+3)}$

$\therefore (x+1)(x-1)(x+3)\neq 0$

$\Rightarrow x\neq -1, x\neq 1, x\neq -3$

$\therefore$ Domain of ' f ' is $R\setminus\{-3,-1,1\}$

5. If $A=\{-2,-1,0,1,2\}$ and $f:A\to B$ is a surjection defined by $f\left(x\right)=x^2+x+1$, then find B

sol. Given $f(x)=x^2+x+1$, $f(0)=0^2+0+1=1$,

$f\left(-2\right)=(-2)^2+(-2)+1=3,\ f\left(-1\right)=(-1)^2+(-1)+1=1,\ f\left(2\right)=2^2+2+1=7,\ f\left(1\right)=1^2+1+1=3$

$\therefore$ Range of $B\{1,3,7\}$

6. find the domain of real valued function $f(x)=\sqrt{9-x^2}$

sol. Given $f(x)=\sqrt{9-x^2}$

but $f(x)\in R\Rightarrow 9-x^2\geq 0\Rightarrow x^2-9\leq 0\Rightarrow (x+3)(x-3)\leq 0\Rightarrow x\in[-3,3]$

$\therefore$ Domain of f is $[-3,3]$

$sol.(fog)\{y\} = f\{g[y]\} = f\left(\frac{y}{\sqrt{1+y^2}}\right) = \frac{\sqrt{1+y^2}}{\sqrt{1-\left(\frac{y}{\sqrt{1+y^2}}\right)^2}} = \frac{\sqrt{1+y^2}}{\sqrt{1-\frac{y^2}{1+y^2}}} = \frac{\sqrt{1+y^2}}{\sqrt{\frac{1+y^2-y^2}{1+y^2}}} = \frac{\sqrt{1+y^2}}{\frac{1}{\sqrt{1+y^2}}} = y$

8. *find the domain of real valued function* $f(x) = \log_e(x^2 - 4x + 3)$

$sol.Given\ f(x) = \log_e(x^2 - 4x + 3)$

$but\ f(x) \in R \Rightarrow x^2 - 4x + 3 > 0 \Rightarrow x^2 - x - 3x + 3 > 0 \Rightarrow (x-1)(x-3) > 0 \Rightarrow x \in (-\infty, 1) \cup (3, \infty)$

$\therefore Domain\ of\ f\ is\ (-\infty, 1) \cup (3, \infty)$

9. *If* $f : Q \to Q$ *is defined by* $f(x) = 5x + 4$ *for all* $x \in Q$ *then find* f^{-1}

$sol.Given\ f(x) = 5x + 4$

$let\ f(x) = y \Rightarrow x = f^{-1}(y) - - - - - - - - (1)$

$\because f(x) = y \Rightarrow 5x + 4 = y \Rightarrow 5x = y - 4 \Rightarrow x = \frac{y-4}{5} - - - - - - - - (2)$

$\therefore f^{-1}(y) = x = \frac{y-4}{5} \Rightarrow f^{-1}(y) = \frac{y-4}{5}$

10. *find the domain of real valued function* $f(x) = \frac{\sqrt{2+x} + \sqrt{2-x}}{x}$

$sol.Given\ f(x) = \frac{\sqrt{2+x} + \sqrt{2-x}}{x}$

$but\ f(x) \in R \Rightarrow 2 + x \ge 0, 2 - x \ge 0, x \ne 0 \Rightarrow -2 \le x \le 2, x \ne 0 \Rightarrow x \in [-2, 2] \setminus \{0\}$

$\therefore Domain\ of\ f\ is\ [-2, 2] \setminus \{0\}$

11. *If* $f : R \to R$ *is defined by* $f(x) = \frac{2x+1}{3}$*, then this function is injuction or not ? justify ?*

$Sol.Given\ f(x) = \frac{2x+1}{3}$

$Suppose\ f(x_1) = f(x_2) \Rightarrow \frac{2x_1+1}{3} = \frac{2x_2+1}{3} \Rightarrow 2x_1 + 1 = 2x_2 + 1 \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2$

$\therefore f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \quad \therefore f$ *is injective function*

12. *If the function f is defined by* $f(x) = \begin{cases} 3x - 2, & x > 3 \\ x^2 - 2, & -2 \le x \le 2. \\ 2x + 1, & x < -3 \end{cases}$

Then find the values of $f(4), f(2.5)$

$sol.If\ x = 4,\ f(x) = 3x - 2$

$f(4) = 3(4) - 2 = 12 - 2 = 10$

$x = 2.5, f(x) = not\ define$

13. *Find the domain of the real valued function* $f(x) = \frac{1}{\sqrt{1-x^2}}$

$sol.Given\ f(x) = \frac{1}{\sqrt{1-x^2}}$

$\therefore 1 - x^2 > 0 \Rightarrow x^2 - 1 < 0 \Rightarrow (x+1)(x-1) < 0 \quad \Rightarrow x \in (-1, 1)$

$\therefore Domain\ of\ 'f'\ is\ (-1, 1)$

Lower triangular matrices : A square matrices $A=[a_{ij}]$ is said to be an lower triangular matrices if $a_{ij}=0$ when $i<j$

14. *Define skew – symmetric matrix, If* $A=\begin{bmatrix}0&2&1\\-2&0&-2\\-1&x&0\end{bmatrix}$ *is a skew – symmetric matrix, find 'x'?*

sol. *skew – symmetric matrix* : *a matrix 'A' is said to be skew – symmetric matrix if* $A^T=-A$

$$\text{Given } A=\begin{bmatrix}0&2&1\\-2&0&-2\\-1&x&0\end{bmatrix}\Rightarrow A^T=\begin{bmatrix}0&-2&-1\\2&0&x\\1&-2&0\end{bmatrix}$$

$$\text{If } A \text{ is a skew – symmetric matrix}, A^T=-A\Rightarrow\begin{bmatrix}0&-2&-1\\2&0&x\\1&-2&0\end{bmatrix}=-\begin{bmatrix}0&2&1\\-2&0&-2\\-1&x&0\end{bmatrix}\Rightarrow x=-(-2)\Rightarrow x=2$$

15. *If* $A=\begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}$ then *find* A^4?

$$\text{Sol. Given } A=\begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}\Rightarrow A=3\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\Rightarrow A=3I\Rightarrow A^4=(3I)^4\Rightarrow A^4=81I\Rightarrow A^4=\begin{bmatrix}81&0&0\\0&81&0\\0&0&81\end{bmatrix}$$

16. *If* $A=\begin{bmatrix}3&2&-1\\2&-2&0\\1&3&1\end{bmatrix}, B=\begin{bmatrix}-3&-1&0\\2&1&3\\4&-1&2\end{bmatrix}$ and X=A+B then *find X*?

$$\text{Sol. Given } A=\begin{bmatrix}3&2&-1\\2&-2&0\\1&3&1\end{bmatrix}, B=\begin{bmatrix}-3&-1&0\\2&1&3\\4&-1&2\end{bmatrix} \text{ and}$$

$$X=A+B\Rightarrow X=\begin{bmatrix}3&2&-1\\2&-2&0\\1&3&1\end{bmatrix}+\begin{bmatrix}-3&-1&0\\2&1&3\\4&-1&2\end{bmatrix}\Rightarrow X=\begin{bmatrix}3-3&2-1&-1+0\\2+2&-2+1&0+3\\1+4&3-1&1+2\end{bmatrix}=\begin{bmatrix}0&1&-1\\4&-1&3\\5&2&3\end{bmatrix}$$

17. *Find the adjoint and inverse of the matrix* $\begin{pmatrix}2&-3\\4&6\end{pmatrix}$

$$\text{Sol. if } A=\begin{pmatrix}a&b\\c&d\end{pmatrix} \text{ then } A^{-1}=\frac{1}{ad-bc}\begin{pmatrix}d&-b\\-c&a\end{pmatrix}, adj\,A=\begin{pmatrix}d&-b\\-c&a\end{pmatrix}$$

$$\text{Given } A=\begin{pmatrix}2&-3\\4&6\end{pmatrix}\Rightarrow|A|=2.6-(-3).4\Rightarrow|A|=12+12=24$$

$$\text{Adjoint } A=\begin{pmatrix}6&3\\-4&2\end{pmatrix}$$

$$\therefore A^{-1}=\frac{adj\,A}{|A|}\Rightarrow A^{-1}=\frac{1}{24}\begin{pmatrix}6&3\\-4&2\end{pmatrix}$$

18. *If* $A=\begin{pmatrix}2&4\\-1&k\end{pmatrix}$ *and* $A^2=0$, then *find the value of k*?

$$\text{Sol. Given } A=\begin{pmatrix}2&4\\-1&k\end{pmatrix} \text{ and } A^2=0$$

$$\therefore AA=0$$

$$\Rightarrow\begin{pmatrix}2&4\\-1&k\end{pmatrix}\begin{pmatrix}2&4\\-1&k\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}4-4&8+4k\\-2-k&-4+k^2\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}$$

$$\Rightarrow 8+4k=0\Rightarrow k=-2$$

$Sol. Given\ A = \begin{pmatrix} \cdots & \cdots \\ 0 & 1 \end{pmatrix} \Rightarrow A^T = \begin{pmatrix} \cdots & \cdots \\ 2 & 1 \end{pmatrix}$

$$\Rightarrow AA^T = \begin{pmatrix} -1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1+4 & 0+2 \\ 0+2 & 0+1 \end{pmatrix} = \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}$$

sol. <u>*Trace*</u> *: sum of the elements in the principal diagonal of a square matrix*

$$Given\ A = \begin{bmatrix} 1 & 3 & -5 \\ 2 & -1 & 5 \\ 2 & 0 & 1 \end{bmatrix}, \quad Tra(A) = 1 + (-1) + 1 = 1$$

21. *Construct a* $3X2$ *matrix whose elements are defined by* $a_{ij} = \frac{1}{2}|\mathrm{i} - 3\mathrm{j}|$

$$Sol. Given\ that\ 3X2\ matrix\ is\ A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}_{3\times 2}$$

$$Now\ a_{ij} = \frac{1}{2}|\mathrm{i} - 3\mathrm{j}|, \forall \mathrm{i} = 1,2,3,\ \mathrm{j} = 1,2$$

$a_{11} = \frac{1}{2}|1-(3x1)| = \frac{1}{2}|1-3| = \frac{1}{2}|-2| = 1$ $\qquad a_{12} = \frac{1}{2}|1-(3x2)| = \frac{1}{2}|1-6| = \frac{1}{2}|-5| = \frac{5}{2}$

$a_{21} = \frac{1}{2}|2-(3x1)| = \frac{1}{2}|2-3| = \frac{1}{2}|-1| = \frac{1}{2}$ $\qquad a_{22} = \frac{1}{2}|2-(3x2)| = \frac{1}{2}|2-6| = \frac{1}{2}|-4| = 2$

$a_{31} = \frac{1}{2}|3-(3x1)| = \frac{1}{2}|3-3| = \frac{1}{2}|0| = 0$ $\qquad a_{32} = \frac{1}{2}|3-(3x2)| = \frac{1}{2}|3-6| = \frac{1}{2}|-3| = \frac{3}{2}$

$$\therefore the\ required\ matrix\ is\ A = \begin{bmatrix} 1 & 5/2 \\ 1/2 & 2 \\ 0 & 3/2 \end{bmatrix}_{3\times 2}$$

21.1. *If* $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & -6 & x \end{bmatrix}$ *and det A = 45, then find 'x'*

$$sol. Given\ A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & -6 & x \end{bmatrix}$$

$\det A = 45 \qquad \Rightarrow 1(3x+24) = 45 \qquad \Rightarrow x = \frac{21}{3} = 7$

22. *Find the rank of* $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

$$Sol. Let\ A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0\ [\because 2\ rows\ are\ identical]$$

$$Take\ a\ 2x2\ \min or, \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0 \quad [\because 2\ rows\ are\ identical]$$

$$\therefore Rank\ of\ A = 1$$

23. *If* $\begin{bmatrix} x-3 & 2y-8 \\ z+2 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ -2 & a-4 \end{bmatrix}$, *find x, y, z and a*

$sol. x-3 = 5 \Rightarrow x = 8, \quad 2y-8 = 2 \Rightarrow y = 5, \quad z+2 = -2 \Rightarrow z = -4, \quad a-4 = 6 \Rightarrow a = 10$

24.Define linear combination of vectors?

Sol. Let $\overrightarrow{a_1}, \overrightarrow{a_2}, \overrightarrow{a_3} ------- \overrightarrow{a_n}$ are vectors and $x_1, x_2, x_3 ------ x_n$ are scalars. Then the vectors $x_1\overrightarrow{a_1} + x_2\overrightarrow{a_2} + x_3\overrightarrow{a_3} + ------- + x_n\overrightarrow{a_n}$ is called a linear combination of vectors where $A=\begin{pmatrix}1 & -1 & 1\\ 1 & 2 & -1\\ 2 & 1 & 3\end{pmatrix}, X=\begin{pmatrix}x\\ y\\ z\end{pmatrix}$

$$|A|=\begin{vmatrix}1 & -1 & 1\\ 1 & 2 & -1\\ 2 & 1 & 3\end{vmatrix}=1(6+1)-(-1)(3+2)+1(1-4)=9\neq 0$$

$\therefore |A|\neq 0 \;\; \therefore Rank\ of\ A=3$

$\therefore$ the given system of equations have trivial solution only

$\therefore x=y=z=0$

26. *Define symmetric matrix, If* $A=\begin{bmatrix}-1 & 2 & 3\\ 2 & 5 & 6\\ 3 & x & 7\end{bmatrix}$ *is a symmetric matrix, find 'x'?*

sol. symmetric matrix : a matrix 'A' is said to be symmetric matrix if $A^T=A$

Given $A=\begin{bmatrix}-1 & 2 & 3\\ 2 & 5 & 6\\ 3 & x & 7\end{bmatrix} \Rightarrow A^T=\begin{bmatrix}-1 & 2 & 3\\ 2 & 5 & x\\ 3 & 6 & 7\end{bmatrix}$

If A is a symmetric matrix, $A^T=A\Rightarrow \begin{bmatrix}-1 & 2 & 3\\ 2 & 5 & 6\\ 3 & x & 7\end{bmatrix}=\begin{bmatrix}-1 & 2 & 3\\ 2 & 5 & x\\ 3 & 6 & 7\end{bmatrix}\Rightarrow x=6$

27. *Find the area of parallelogram whose diagonals are* $3i+j-2k$ *and* $i-3j+4k$

Sol. Let $a=3i+j-2k, b=i-3j+4k$

$$axb=\begin{vmatrix}i & j & k\\ 3 & 1 & -2\\ 1 & -3 & 4\end{vmatrix}=i(4-6)-j(12+2)+k(-9-1)=-2i-14j-10k$$

$$|axb|=\sqrt{(-2)^2+(-14)^2+(-10)^2}=\sqrt{300}=10\sqrt{3}$$

$\therefore$ the area of parallelogram whose diagonals are $\vec{a},\vec{b}=\frac{1}{2}|axb|=\frac{1}{2}.10\sqrt{3}=5\sqrt{3}$ sq.units

28. *Show that the points whose position vectors* $-2a+3b+5c,\ a+2b+3c,\ 7a-c$ *are collinear*

sol. consider $\begin{vmatrix}-2 & 3 & 5\\ 1 & 2 & 3\\ 7 & 0 & -1\end{vmatrix}\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

$$=[-2(-2-0)-3(-1-21)+5(0-14)]\left[\vec{a}\,\vec{b}\,\vec{c}\right]=(0)\left[\vec{a}\,\vec{b}\,\vec{c}\right]=0$$

$\therefore$ given points are collinear

29. *Find the unit vector in the direction of* $2\bar{i}+3\bar{j}+\bar{k}$

sol. Given $\vec{a}=2\bar{i}+3\bar{j}+\bar{k}$ then $|\vec{a}|=\sqrt{2^2+3^2+1^2}=\sqrt{4+9+1}=\sqrt{14}$

$\therefore$ unit vector in the direction of $\vec{a}=\frac{\vec{a}}{|\vec{a}|}=\frac{2\bar{i}+3\bar{j}+\bar{k}}{\sqrt{14}}$

we have $\frac{2}{4}=\frac{3}{m}=\frac{1}{n}\Rightarrow\frac{1}{2}=\frac{3}{m}=\frac{1}{n}\Rightarrow m=10,n=2$

31. *Find the vector equation of the line* passing through *the points* $\vec{a}=2\bar{i}+3\bar{j}+\bar{k}$ *and parallel to the vector* $\vec{b}=4i-2j+3k$

sol.The vector equation of the line passing through *the points* $\vec{a}$ *and parallel to the vector* $\vec{b}$ *is* $\vec{r}=\vec{a}+t\vec{b}$

given $\vec{a}=2\bar{i}+3\bar{j}+\bar{k}$ *and* $\vec{b}=4i-2j+3k$

$\therefore$ *The line* $\vec{r}=2\bar{i}+3\bar{j}+\bar{k}+t(4i-2j+3k)$

32. *If the position vectors of the points A, B and C are* $-2i+j-k$, $-4i+2j+2k$ *and* $6i-3j-13k$ *respectively and* $\overrightarrow{AB}=\lambda\overrightarrow{AC}$, *then find* λ?

Sol.Given $\overrightarrow{OA}=-2i+j-k,\overrightarrow{OB}=-4i+2j+2k$ *and* $\overrightarrow{OC}=6i-3j-13k$

$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=(-4i+2j+2k)-(-2i+j-k)=-2i+j+3k$

$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=(6i-3j-13k)-(-2i+j-k)=8i-4j+12k$

Given $\overrightarrow{AB}=\lambda\overrightarrow{AC}\Rightarrow(-2i+j+3k)=\lambda(8i-4j+12k)\Rightarrow 8\lambda=-2\Rightarrow\lambda=\frac{-1}{4}$

33. *Find the area of the parallelogram having* $2\bar{i}-3\bar{j}$ *and* $3\bar{i}-\bar{k}$ *as adjacent sides*

sol.The area of the parallelogram $=|\vec{a}\times\vec{b}|$

Given $\vec{a}=2\bar{i}-3\bar{j}$ *and* $\vec{b}=3\bar{i}-\bar{k}$

now $\vec{a}\times\vec{b}=\begin{vmatrix} i & j & k \\ 2 & -3 & 0 \\ 3 & 0 & -1 \end{vmatrix}=i(3-0)-j(-2-0)+k(0+9)=3i+2j+9k$

$|\vec{a}\times\vec{b}|=\sqrt{3^2+2^2+9^2}=\sqrt{9+4+81}=\sqrt{94}$ *sq.units*

34. *Find the area of the parallelogram having* $2\bar{j}-\bar{k}$ *and* $-\bar{i}+\bar{k}$ *as adjacent sides*

sol.The area of the parallelogram $=|\vec{a}\times\vec{b}|$

Given $\vec{a}=2\bar{j}-\bar{k}$ *and* $\vec{b}=-\bar{i}+\bar{k}$

now $\vec{a}\times\vec{b}=\begin{vmatrix} i & j & k \\ 0 & 2 & -1 \\ -1 & 0 & 1 \end{vmatrix}=i(2-0)-j(0-3)+k(0+2)=2i+j+2k$

$|\vec{a}\times\vec{b}|=\sqrt{2^2+1^2+2^2}=\sqrt{4+1+4}=\sqrt{9}=3$ *sq.units*

35. *If* α,β,γ *are the angles made by the vector* $3i-6j+2k$ *with the positive direction of coordinate axis, then find* $\cos\alpha,\cos\beta,\cos\gamma$?

Sol.Given $\vec{a}=3i-6j+2k,\Rightarrow|\vec{a}|=\sqrt{3^2+(-6)^2+2^2}=\sqrt{49}=7$

then $\cos\alpha=\frac{3}{7},\cos\beta=\frac{-6}{7},\cos\gamma=\frac{2}{7}$

36. *Find the angle between the vectors* $\vec{a}=i+2j+3k$ *and* $\vec{b}=3i-j+2k$

sol.We know that $\cos\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

given that $\vec{a}=i+2j+3k\Rightarrow|\vec{a}|=\sqrt{1^2+2^2+3^2}=\sqrt{14}$ *and* $\vec{b}=3i-j+2k\Rightarrow|\vec{b}|=\sqrt{3^2+(-1)^2+2^2}=\sqrt{14}$

$\cos\theta=\frac{1(3)+2(-1)+3(2)}{\sqrt{14}\sqrt{14}}=\frac{7}{14}=\frac{1}{2}\Rightarrow\theta=\frac{\pi}{3}$

given $\vec{a} = i - 2j + 5k$, $b = -5j - k$ and $\vec{c} = -3i + 5j$

$\therefore$ The line $\vec{r} = (1-t-s)(\bar{i} - 2\bar{j} + 5\bar{k}) + t(-5\bar{j} - \bar{k}) + s(-3\bar{i} + 5\bar{j})$

38. If $\vec{a} = i - j - k$ and $\vec{b} = 2i - 3j + k$ then find the projection of $\vec{b}$ on $\vec{a}$?

Sol. Given $\vec{a} = i - j - k$ and $\vec{b} = 2i - 3j + k$

$\vec{b}.\vec{a} = (2i - 3j + k).(i - j - k) = 2 + 3 - 1 = 4$

$|\vec{a}| = \sqrt{1^2 + (-1)^2 + (-1)^2} = \sqrt{3} \Rightarrow |\vec{a}|^2 = 3$

$\therefore$ the projection of $\vec{b}$ on $\vec{a} = \dfrac{(\vec{b}.\vec{a})\vec{a}}{|\vec{a}|^2} = \dfrac{4}{3}(i - j - k)$

39. Let $\vec{a} = i + 2j + 3k$ and $\vec{b} = 3i + j$. Find the unit vector in the direction of $\vec{a} + \vec{b}$?

Sol. Given $\vec{a} = i + 2j + 3k$ and $\vec{b} = 3i + j$

$\vec{a} + \vec{b} = (i + 2j + 3k) + (3i + j) = 4i + 3j + 3k$

$|\vec{a} + \vec{b}| = \sqrt{4^2 + 3^2 + 3^2} = \sqrt{34}$

the unit vector in the direction of $\vec{a} + \vec{b} = \dfrac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|} = \dfrac{1}{\sqrt{34}}(4i + 3j + 3k)$

40. If the vectors $\vec{a} = 2\bar{i} + \lambda\bar{j} - \bar{k}$ and $\vec{b} = 4\bar{i} - 2\bar{j} + 2\bar{k}$ are perpendicular to each other, find λ

sol. If $\vec{a} \perp \vec{b} \Leftrightarrow \vec{a}.\vec{b} = 0$

$\Rightarrow 2(4) + \lambda(-2) + (-1)2 = 0 \qquad \Rightarrow \lambda = 3$

41. Find the vector equation of the line joining the points $\vec{a} = 2i + j + 3k$ and $\vec{b} = -4i + 3j - k$

sol. The vector equation of the line joining the points $\vec{a}$ and $\vec{b}$ is $\vec{r} = (1-t)\vec{a} + t\vec{b}$

given $\vec{a} = 2\bar{i} + \bar{j} + 3\bar{k}$ and $\vec{b} = -4i + 3j - k$

$\therefore$ The line $\vec{r} = (1-t)2\bar{i} + \bar{j} + 3\bar{k} + t(-4i + 3j - k)$

42. Find the angle between the planes $r.(2i - j + 2k) = 3$ and $r.(3i + 6j + k) = 4$

Sol. Given planes are $r.(2i - j + 2k) = 3 - - - - - - -(1)$

and $r.(3i + 6j + k) = 4 - - - - - - -(2)$

let θ be the angle between planes (1) and (2)

we know that $\cos\theta = \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{(a_1^2 + b_1^2 + c_1^2)(a_2^2 + b_2^2 + c_2^2)}}$

$\Rightarrow \cos\theta = \dfrac{2.3 + (-1).6 + 2.1}{\sqrt{(2^2 + (-1)^2 + 2^2)(3^2 + 6^2 + 1^2)}}$

$\Rightarrow \cos\theta = \dfrac{6 - 6 + 2}{\sqrt{(4+1+4)(9+36+1)}} \Rightarrow \cos\theta = \dfrac{2}{3\sqrt{46}} \Rightarrow \theta = \cos^{-1}\left(\dfrac{2}{3\sqrt{46}}\right)$

43. If the vectors $\lambda\bar{i} - 3\bar{j} + 5\bar{k}$ and $2\lambda\bar{i} - \lambda\bar{j} - \bar{k}$ are perpendicular to each other, find λ

sol. Given $\vec{a} = \lambda\bar{i} - 3\bar{j} + 5\bar{k}$ and $\vec{b} = 2\lambda\bar{i} - \lambda\bar{j} - \bar{k}$

$\vec{a} \perp \vec{b} \Leftrightarrow \vec{a}.\vec{b} = 0 \qquad \Rightarrow \lambda(2\lambda) + (-3)(-\lambda) + 5(-1) = 0 \quad \Rightarrow 2\lambda^2 + 3\lambda - 5 = 0$

$\Rightarrow (2\lambda + 5)(\lambda - 1) = 0 \qquad \Rightarrow \lambda = 1$ or $\lambda = \dfrac{-5}{2}$

45. *Find the period of the function* $f(x)=\cos(3x+5)+7$

Sol.The period of the function $\cos a\theta$ *is* $\frac{2\pi}{a}$

given $f(x)=\cos(3x+5)+7$

$\therefore$ *the period of* $f(x)=\frac{2\pi}{3}$

46. *If* $\sin\alpha=\frac{1}{\sqrt{10}},\sin\beta=\frac{1}{\sqrt{5}}$ *and* α,β *are acute, then show that* $\alpha+\beta=\frac{\pi}{4}$?

Sol.Given $\sin\alpha=\frac{1}{\sqrt{10}}$ $\qquad \sin\beta=\frac{1}{\sqrt{5}}$

$\tan\alpha=\frac{1}{3}$ $\qquad \tan\beta=\frac{1}{2}$

$$\therefore \tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}.\frac{1}{2}}=\frac{\frac{2+3}{6}}{\frac{6-1}{6}}=\frac{5}{5}=1=\tan\frac{\pi}{4}$$

$$\Rightarrow \tan(\alpha+\beta)=\tan\frac{\pi}{4}\Rightarrow\alpha+\beta=\frac{\pi}{4}$$

47. *If* $\sin\theta=\frac{4}{5}$ *and* θ *is not in the first quadrant, find the value of* $\cos\theta$?

Sol. given $\sin\theta=\frac{4}{5}>0$, *and* θ *is not in the first quadrant*

$\therefore\theta$ *lies in second quadrant* $\Rightarrow\cos\theta$ *is negative*

we know that $\cos^2\theta=1-\sin^2\theta$

$$\Rightarrow\cos^2\theta=1-\left(\frac{4}{5}\right)^2=1-\frac{16}{25}=\frac{25-16}{25}=\frac{9}{25}=\left(\frac{3}{5}\right)^2$$

$\therefore\cos\theta=\pm\frac{3}{5}$, *Hence* $\cos\theta=-\frac{3}{5}$

48. Pr*ove that* $\cos 48.\cos 12=\frac{3+\sqrt{5}}{8}$

Sol.L.H.S $=\cos 48.\cos 12$

$=\frac{1}{2}[2\cos 48.\cos 12]$

$=\frac{1}{2}[\cos(48+12)+\cos(48-12)]$

$=\frac{1}{2}[\cos 60+\cos 36]$

$=\frac{1}{2}\left[\frac{1}{2}+\frac{\sqrt{5}+1}{4}\right]=\frac{1}{2}\left[\frac{2+\sqrt{5}+1}{4}\right]=\frac{3+\sqrt{5}}{8}=R.H.S$

49. Pr*ove that* $\frac{\cos 9+\sin 9}{\cos 9-\sin 9}=\cot 36$

$$Sol.Given\ \frac{\cos 9+\sin 9}{\cos 9-\sin 9}=\frac{\cos 9\left(1+\frac{\sin 9}{\cos 9}\right)}{\cos 9\left(1-\frac{\sin 9}{\cos 9}\right)}=\frac{\tan 45+\tan 9}{1-\tan 45.\tan 9}=\tan(45+9)=\tan 54=\tan(90-36)=\cot(36)$$

50. *Find the period of the function* $f(x)=\tan(x+4x+9x+------+n^2x)$

Sol.The period of the function $\tan a\theta$ *is* $\frac{\pi}{a}$

given $f(x)=\tan(x+4x+9x+------+n^2x)$

$$f(x)=\tan(1+2^2+3^2+------+n^2)x=\tan\left(\frac{n(n+1)(2n+1)}{6}\right)x$$

$$\therefore \text{the period of } f(x)=\frac{\pi}{\frac{n(n+1)(2n+1)}{6}}=\frac{6\pi}{n(n+1)(2n+1)}$$

we know that $\sin^2\theta = 1-\cos^2\theta$

$\Rightarrow \sin^2\theta = 1-t^2 \Rightarrow \sin^2\theta = \pm\sqrt{1-t^2}$

$\therefore \sin\theta = -\sqrt{1-t^2}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = -\dfrac{\sqrt{1-t^2}}{t}$

52. *Find the maximum and minimum value of* $13\cos x + 3\sqrt{3}\sin x - 4$

Sol. Given $13\cos x + 3\sqrt{3}\sin x - 4$

Here $a = 13, b = 3\sqrt{3}, c = -4$

$\max.value = c + \sqrt{a^2+b^2} = -4 + \sqrt{13^2 + (3\sqrt{3})^2} = -4 + \sqrt{169+27} = -4 + \sqrt{196} = -4 + 14 = 10$

$\min.value = c - \sqrt{a^2+b^2} = -4 - \sqrt{13^2 + (3\sqrt{3})^2} = -4 - \sqrt{169+27} = -4 - \sqrt{196} = -4 - 14 = -18$

53. *Find the maximum and minimum value of* $3\cos x + 4\sin x$

Sol. Given $3\cos x + 4\sin x$

Here $a = 3, b = 4, c = 0$

$\max.value = c + \sqrt{a^2+b^2} = 0 + \sqrt{3^2+4^2} = +\sqrt{9+16} = +\sqrt{25} = 5$

$\min.value = c - \sqrt{a^2+b^2} = 0 - \sqrt{3^2+4^2} = -\sqrt{9+16} = -\sqrt{25} = -5$

58. *Find a sine function whose period as* $\dfrac{2}{3}$

Sol. The period of $\sin ax$ *is* $\dfrac{2\pi}{|a|}$

given $\dfrac{2\pi}{|a|} = \dfrac{2}{3} \Rightarrow |a| = 3\pi \Rightarrow a = \pm 3\pi$

$\therefore$ *the sine function* $= \sin(\pm 3\pi)x$

55. *If* $A+B = \dfrac{\pi}{4}$ *then prove that*

$(1+\tan A)(1+\tan B) = 2$

Sol. Given $A+B = \dfrac{\pi}{4}$

$\Rightarrow \tan(A+B) = \tan\left(\dfrac{\pi}{4}\right)$

$\Rightarrow \dfrac{\tan A + \tan B}{1-\tan A.\tan B} = 1$

$\Rightarrow \tan A + \tan B = 1 - \tan A.\tan B$

$\Rightarrow \tan A + \tan B + \tan A.\tan B = 1$

Adding 1 on both side, we have

$1 + \tan A + \tan B + \tan A.\tan B = 1+1$

$\Rightarrow (1+\tan A) + \tan B(1+\tan A) = 2$

$\Rightarrow (1+\tan A)(1+\tan B) = 2$

54. *Eliminate* θ *from* $x = a\cos^3\theta, y = b\sin^3\theta$

Sol. Given $x = a\cos^3\theta, y = b\sin^3\theta$

$\Rightarrow \dfrac{x}{a} = \cos^3\theta, \dfrac{y}{b} = \sin^3\theta$

$\Rightarrow \left(\dfrac{x}{a}\right)^{\frac{2}{3}} = \left(\cos^3\theta\right)^{\frac{2}{3}}, \left(\dfrac{y}{b}\right)^{\frac{2}{3}} = \left(\sin^3\theta\right)^{\frac{2}{3}}$

$\Rightarrow \left(\dfrac{x}{a}\right)^{\frac{2}{3}} = \cos^2\theta, \left(\dfrac{y}{b}\right)^{\frac{2}{3}} = \sin^2\theta$

$\Rightarrow \left(\dfrac{x}{a}\right)^{\frac{2}{3}} + \left(\dfrac{y}{b}\right)^{\frac{2}{3}} = \cos^2\theta + \sin^2\theta = 1$

$\Rightarrow \left(\dfrac{x}{a}\right)^{\frac{2}{3}} + \left(\dfrac{y}{b}\right)^{\frac{2}{3}} = 1$

56. *Prove that* $\tan 70 - \tan 20 = 2\tan 50$

Sol. We know $70 - 20 = 50$

$\Rightarrow \tan(70-20) = \tan 50$

$\Rightarrow \dfrac{\tan 70 - \tan 20}{1+\tan 70.\tan 20} = \tan 50$

$\Rightarrow \dfrac{\tan 70 - \tan 20}{1+\tan 70.\cot 70} = \tan 50$

$\Rightarrow \dfrac{\tan 70 - \tan 20}{1+1} = \tan 50$

$\Rightarrow \tan 70 - \tan 20 = 2\tan 50$

57. *If* $\tan 20 = \lambda$, *then show that* $\dfrac{\tan 160 - \tan 110}{1+\tan 160.\tan 110} = \dfrac{1-\lambda^2}{2\lambda}$

Sol. Given $\tan 20 = \lambda \Rightarrow \cot 20 = \dfrac{1}{\lambda}$

$LHS = \dfrac{\tan 160 - \tan 110}{1+\tan 160.\tan 110}$

$= \tan(160-110) = \tan 50 = \tan(90-40) = \cot 40$

$= \cot 2(20) = \dfrac{\cot^2 20 - 1}{2\cot 20} = \dfrac{\left(\dfrac{1}{\lambda}\right)^2 - 1}{2\dfrac{1}{\lambda}} = \dfrac{1-\lambda^2}{2\lambda}$

sol.Given that $\sinh x = \frac{3}{4}$

we know that $\cosh^2 x = 1 + \sinh^2 x$

$$\cosh^2 x = 1 + \left(\frac{3}{4}\right)^2 = 1 + \frac{9}{16} = \frac{25}{16} \Rightarrow \cosh x = \frac{5}{4}$$

$$\cosh(2x) = 1 + 2\sinh^2 x = 1 + 2\left(\frac{9}{16}\right) = 1 + \frac{9}{8} = \frac{17}{8}$$

$$\sinh(2x) = 2\sinh x.\cosh x = 2.\frac{3}{4}.\frac{5}{4} = \frac{15}{8}$$

64. *If* $\cosh x = \frac{5}{2}$, *then find the values of* $\cosh(2x), \sinh(2x)$

Sol.Given $\cosh x = \frac{5}{2}$

we know that $\cosh^2 x - \sinh^2 x = 1$

$$\Rightarrow \sinh^2 x = \cosh^2 x - 1$$

$$\Rightarrow \sinh^2 x = \left(\frac{5}{2}\right)^2 - 1 = \frac{25}{4} - 1 = \frac{21}{4}$$

$$\Rightarrow \sinh x = \frac{\sqrt{21}}{2}$$

now $\cosh(2x) = \cosh^2 x + \sinh^2 x = \frac{25}{4} + \frac{21}{4} = \frac{46}{4} = \frac{23}{2}$

and $\sinh(2x) = 2\sinh x.\cosh x = 2.\frac{\sqrt{21}}{2}.\frac{5}{2} = \frac{5\sqrt{21}}{2}$

65. *P.T* $(\cosh x - \sinh x)^n = \cosh(nx) - \sinh(nx)$

Sol.LHS $= (\cosh x - \sinh x)^n = \left[\frac{e^x + e^{-x}}{2} - \frac{e^x - e^{-x}}{2}\right]^n$

$$\Rightarrow (\cosh x - \sinh x)^n = \left[\frac{e^x + e^{-x} - e^x + e^{-x}}{2}\right]^n$$

$$\Rightarrow (\cosh x - \sinh x)^n = \left[\frac{2e^{-x}}{2}\right] = e^{-x}$$

$RHS = \cosh(nx) - \sinh(nx)$

$$= \frac{e^{nx} + e^{-nx}}{2} - \frac{e^{nx} - e^{-nx}}{2}$$

$$= \frac{e^{nx} + e^{-nx} - e^{nx} + e^{-nx}}{2}$$

$$= \frac{2e^{-nx}}{2} = e^{-nx}$$

$L.H.S = R.H.S$

$$\therefore (\cosh x - \sinh x)^n = \cosh(nx) - \sinh(nx)$$

59. *If* $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$,

prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

Sol.Given $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$$\Rightarrow \sin\theta = \sqrt{2}\cos\theta - \cos\theta$$

$$\Rightarrow \sin\theta = \left(\sqrt{2} - 1\right)\cos\theta$$

multiplying with $\left(\sqrt{2} + 1\right)$ *on both side*

$$\Rightarrow \left(\sqrt{2}+1\right)\sin\theta = \left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)\cos\theta$$

$$\Rightarrow \sqrt{2}\sin\theta + \sin\theta = (2-1)\cos\theta$$

$$\Rightarrow \sqrt{2}\sin\theta = \cos\theta - \sin\theta$$

60. *If* $\sin\alpha = \frac{3}{5}$, *where* $\frac{\pi}{2} < \alpha < \pi$,

Evaluate $\cos 3\alpha$

Sol.given $\sin\alpha = \frac{3}{5}$ *and* $\frac{\pi}{2} < \alpha < \pi$

$\therefore \alpha$ *lies in second quadrant*

we know $\cos^2\alpha = 1 - \sin^2\alpha$

$$\cos^2\alpha = 1 - \frac{9}{25} = \frac{16}{25}$$

$$\therefore \cos\alpha = \pm\frac{4}{5} \Rightarrow \cos\alpha = -\frac{4}{5}$$

now $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$

$$\cos 3\alpha = 4\left(\frac{-4}{5}\right)^3 - 3\left(\frac{-4}{5}\right)$$

$$\cos 3\alpha = 4\left(\frac{-64}{125}\right) + \left(\frac{12}{5}\right)$$

$$\cos 3\alpha = \frac{-256+300}{125} = \frac{44}{125}$$

61. *Show that* $\tan h^{-1}\left(\frac{1}{2}\right) = \frac{1}{2}\log_e 3$

sol.We know that $\tanh^{-1}(x) = \frac{1}{2}\log_e\left(\frac{1+x}{1-x}\right)$

$$\therefore \tanh^{-1}\left(\frac{1}{2}\right) = \frac{1}{2}\log_e\left(\frac{1+1/2}{1-1/2}\right) = \frac{1}{2}\log_e 3$$

62. *If* $\sinh x = 3$ *show that* $x = \log_e\left(3+\sqrt{10}\right)$

sol.We know that $\sinh^{-1} x = \log_e\left(x + \sqrt{x^2+1}\right)$

given that $\sinh x = 3 \Rightarrow x = \sinh^{-1} 3$

$$\Rightarrow x = \log_e\left(3+\sqrt{3^2+1}\right) \Rightarrow x = \log_e\left(3+\sqrt{10}\right)$$