1. *Transform the equations into* (a) *slope – intercept from* (b) *intercept form and* (c) normal form

$i) 3x+4y+12=0 \quad ii) x+y+1=0 \quad iii) \sqrt{3}x+y=4 \quad iv) 4x-3y+12=0$

sol. (a) <u>*slope – intercept from*</u>

$$\text{given line } 3x+4y+12=0 \Rightarrow 4y=-3x-12 \Rightarrow y=\frac{-3}{4}x-\frac{12}{4} \Rightarrow y=\frac{-3}{4}x-3$$

(b) <u>*intercept form*</u>

$$\text{given line } 3x+4y+12=0 \Rightarrow 3x+4y=-12 \Rightarrow \frac{3x}{-12}+\frac{4y}{-12}=1 \Rightarrow \frac{x}{-4}+\frac{y}{-3}=1$$

(c) <u>Normal form</u>

$$\text{given line } 3x+4y+12=0 \Rightarrow 3x+4y=-12$$

$$\text{divide with } \sqrt{3^2+4^2}=\sqrt{9+16}=\sqrt{25}=5 \text{ on both side}$$

$$\Rightarrow \frac{3x}{5}+\frac{4y}{5}=\frac{-12}{5} \Rightarrow \frac{-3x}{5}+\frac{(-4y)}{5}=\frac{12}{5}$$

$$\therefore \text{The equation of the line in normal form is } x\cos\alpha+y\sin\alpha=p \text{ where } \cos\alpha=\frac{-3}{5}, \sin\alpha=\frac{-4}{5}, p=\frac{12}{5}$$

2. *Find the value of* P, *if the straight lines* $x+p=0,\ y+2=0,\ 3x+2y+5=0$ *are concurrent*

$$sol. \text{given } x+p=0 \Rightarrow x=-p \text{ ------(1)}$$

$$y+2=0 \Rightarrow y=-2 \text{ --------(2)}$$

$$3x+2y+5=0 \text{ -------------(3)}$$

$$\text{from (1),(2),(3), } 3(-p)+2(-2)+5=0 \Rightarrow -3p-4+5=0 \Rightarrow p=\frac{1}{3}$$

3. *Find the value of 'y', if the line joining the points* $(3,y)$ *and* $(2,7)$ *is parallel to the line joining the points* $(-1,4),(0,6)$

$$Sol. \text{Let } A=(3,y),\ B=(2,7),\ C=(-1,4),\ D=(0,6)$$

$$\text{slope of } AB=\frac{y_2-y_1}{x_2-x_1}=\frac{7-y}{2-3}=\frac{7-y}{-1}=y-7$$

$$\text{slope of } CD=\frac{y_2-y_1}{x_2-x_1}=\frac{6-4}{0+1}=\frac{2}{1}=2$$

$$\text{since } AB \| CD, \text{ slope of } AB=\text{slope of } CD$$

$$\Rightarrow y-7=2 \Rightarrow y=9$$

4. *Find the equation of the straight line passing through* $(-2,4)$ *and making non – zero intercepts whose sum is zero*

$$sol. \text{The equation of the line in the intercept form } \frac{x}{a}+\frac{y}{b}=1 \text{ --------(1)}$$

$$\text{given that the sum of the intercept, } a+b=0 \Rightarrow b=-a$$

$$\text{from equation (1), the line is } \frac{x}{a}+\frac{y}{-a}=1 \Rightarrow x-y=a \text{ --------(2)}$$

$$\text{the equation (2) passing through } (-2,4)$$

$$ie, -2-4=a \Rightarrow a=-6$$

$$\therefore \text{the equation of the line is } x-y=-6$$

given lines are $6x-10y+3=0$ and $kx-5y+8=0$

$\Rightarrow \frac{6}{k}=\frac{-10}{5} \Rightarrow k=3$

6. Find the slope of the lines $x+y=0$ and $x-y=0$

Sol. Given line $x+y=0 \Rightarrow y=-x \Rightarrow y=(-1)x \quad \therefore$ slope $m=-1$

also given line $x-y=0 \Rightarrow y=x \Rightarrow y=(1)x \quad \therefore$ slope $m=1$

7. Find the value of 'x' if the slope of the line passing through $(2,5)$ and $(x,3)$ is 2

Sol. Let $A(x_1,y_1)=(2,5)$, $B(x_2,y_2)=(x,3)$

slope of $AB=2$

$\Rightarrow \frac{y_2-y_1}{x_2-x_1}=2 \quad \Rightarrow \frac{3-5}{x-2}=2 \quad \Rightarrow 2x-4=-2 \quad \Rightarrow x=1$

8. i) Find the length of the perpendicular from the point $(-2,-3)$ to the straight line $5x-2y+4=0$

ii) Find the length of the perpendicular from the point $(3,4)$ to the straight line $3x-4y+10=0$

Sol. The length of the perpendicular from the point (α,β) to the straight line $ax+by+c=0$ is, $d=\frac{|a\alpha+b\beta+c|}{\sqrt{a^2+b^2}}$

the length of the perpendicular from the point $(-2,-3)$ to the straight line $5x-2y+4=0$, $d=\frac{|5(-2)-2(-3)+4|}{\sqrt{5^2+(-2)^2}}$

$\Rightarrow d=\frac{|-10+6+4|}{\sqrt{5^2+(-2)^2}} \Rightarrow d=\frac{0}{\sqrt{5^2+(-2)^2}} \Rightarrow d=0$

9. Find the value of p, if the straight line $3x+7y-1=0$ and $7x-py+3=0$ are mutually perpendicular.

Sol. If the lines $a_1x+b_1y+c_1=0, a_2x+b_2y+c_2=0$ are perpendicular $\Leftrightarrow a_1a_2+b_1b_2=0$

given lines $3x+7y-1=0$ and $7x-py+3=0$

$\Rightarrow 3(7)+7(-p)=0 \quad \Rightarrow p=3$

10. Find the equation of the straight line perpendicular to the line $5x-3y+1=0$ and passing through the point $(4,-3)$

Sol. The equation of the straight line perpendicular to the line $ax+by+c=0$ is $bx-ay+k=0$

the given line $5x-3y+1=0$ ----------(1)

the perpendicular line is $-3x-5y+k=0 \Rightarrow 3x+5y-k=0$ ------(2)

the equation (2) passing through $(4,-3)$ ie, $3(4)+5(-3)-k=0 \Rightarrow k=-3$

$\therefore$ the required equation of the line is $3x+5y+3=0$

11. Find the equation of the straight line passing through $(-4,5)$ and cutting off equal non-zero intercepts

sol. The equation of the line in the intercept form $\frac{x}{a}+\frac{y}{b}=1$ ---------(1)

given that the sum of the intercept, $a+b=0 \Rightarrow b=-a$

from equation (1), the line is $\frac{x}{a}+\frac{y}{-a}=1 \Rightarrow x-y=a$ ---------(2)

the equation (2) passing through $(-2,4)$

ie, $-2-4=a \Rightarrow a=-6$

$\therefore$ the equation of the line is $x-y=-6$

$$(y-y_1)=\left(\frac{y_2-y_1}{x_2-x_1}\right)(x-x_1)$$

$$\Rightarrow (y-2at_1)=\left(\frac{2at_2-2at_1}{at_2^2-at_1^2}\right)(x-at_1^2)$$

$$\Rightarrow (y-2at_1)=\left(\frac{2a(t_2-t_1)}{a(t_2+t_1)(t_2-t_1)}\right)(x-at_1^2)$$

$$\Rightarrow (t_2+t_1)(y-2at_1)=2(x-at_1^2)$$

$$\Rightarrow (t_2+t_1)y-2at_1(t_2+t_1)=2x-2at_1^2$$

$$\Rightarrow (t_2+t_1)y-2at_1t_2-2at_1^2=2x-2at_1^2$$

$$\Rightarrow 2x-(t_2+t_1)y+2at_1t_2=0$$

13. *Find the area of triangle formed by the line* $3x-4y+12=0$ *with co-ordinate axes*

Sol. The area of triangle formed by the line $ax+by+c=0$ *with co-ordinate axes is* $\frac{C^2}{2|ab|}$

the given line is $3x-4y+12=0 \quad \Rightarrow a=3, b=-4, c=12$

$\therefore$ *area of triangle* $=\frac{12^2}{2|3.(-4)|}=\frac{144}{24}=6$ *sq.units*

14. *Find the value of 'p', if the lines* $4x-3y-7=0$, $2x+py+2=0$, *and* $6x+5y-1=0$ *are concurrent*

Sol. Given lines $4x-3y-7=0$, $2x+py+2=0$, *and* $6x+5y-1=0$ *are concurrent*

$$\Rightarrow \begin{vmatrix} 4 & -3 & -7 \\ 2 & p & 2 \\ 6 & 5 & -1 \end{vmatrix}=0$$

$$\Rightarrow 4(-p-10)+3(-2-12)-7(10-6p)=0$$

$$\Rightarrow -4p-40-6-36-70+42p=0$$

$$\Rightarrow p=4$$

15. *Find the ratio in which the straight line* $2x+3y=5$ *divide the line joining the points* $(0,0)$ *and* $(-2,1)$

Sol. Let $L\equiv 2x+3y-5=0$, $A(x_1,y_1)=(0,0), B(x_2,y_2)=(-2,1)$

now $L_{11}=2(0)+3(0)-5=-5 \quad L_{22}=2(-2)+3(1)-5=-6$

$\therefore$ *the ratio is* $-L_{11}:L_{22}=-(-5):-6=5:-6$

16. *Find the distance between the parallel straight lines* $3x+4y-3=0$, $6x+8y-1=0$

sol. The distance between the parallel straight lines $ax+by+c_1=0, ax+by+c_2=0$ *is* $\frac{|c_2-c_1|}{\sqrt{a^2+b^2}}$

given lines $3x+4y-3=0 \Rightarrow 2(3x+4y-3=0)\Rightarrow 6x+8y-6=0$ *and* $6x+8y-1=0$

$\therefore$ *The distance between the parallel lines is* $\frac{|-1-(-6)|}{\sqrt{6^2+8^2}}=\frac{|-1+6|}{\sqrt{36+64}}=\frac{5}{10}=\frac{1}{2}$

$the\ parallel\ line\ is\ 2x+3y+k=0 ------(2)$

$the\ equation\ (2)\ passing\ through\ (5,4)\ ie, 2(5)+3(4)+k=0 \Rightarrow k=-22$

$\therefore\ the\ required\ equation\ of\ the\ line\ is\ 2x+3y-22=0$

18. *Find the ratio in which XZ – plane divides the line joining* $A\ (-2,\ 3,\ 4)$ *and* $B\ (1,2,3)$

sol. The ratio in which XZ – plane divides the line joining $A\ (x_1, y_1, z_1)$ *and* $B\ (x_2, y_2, z_2)$ *is* $-y_1 : y_2$

given points $A\ (-2,\ 3,\ 4)$ *and* $B\ (1,2,3)$

the ratio is $-3:2$

19. *Find the fourth vertex of the parallelogram whose consecutive vertices are* $(2,4,-1), (3,6,-1)$ *and* $(4,5,1)$

Sol. let $A=(2,4,-1), B=(3,6,-1), C=(4,5,1)$ *and* $D=(x,y,z)$

$\because$ *ABCD is a parallelogram*

mid point of AC = mid point of BD

$$\Rightarrow \left(\frac{2+4}{2}, \frac{4+5}{2}, \frac{-1+1}{2}\right) = \left(\frac{3+x}{2}, \frac{6+y}{2}, \frac{-1+z}{2}\right)$$

$$\Rightarrow \left(3, \frac{9}{2}, 0\right) = \left(\frac{3+x}{2}, \frac{6+y}{2}, \frac{-1+z}{2}\right)$$

$$\Rightarrow \frac{3+x}{2}=3, \quad \frac{6+y}{2}=\frac{9}{2}, \quad \frac{-1+z}{2}=0$$

$$\Rightarrow 3+x=6, \quad 6+y=9, \quad -1+z=0$$

$$\Rightarrow x=3, \quad y=3, \quad z=1$$

$\therefore$ *the fourth vertex is* $D=(3,3,1)$

20. *i) Show that the points* $(5,4,2), (6,2,-1)$ *and* $(8,-2,-7)$ *are collinear*

ii) Show that the points are $(1,2,3), (7,0,1)$ *and* $(-2,3,4)$ *are collinear*

Sol. Let $A=(5,4,2)\ B=(6,2,-1)\ C=(8,-2,-7)$

$$now\ AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(6-5)^2+(2-4)^2+(-1-2)^2} = \sqrt{1+4+9} = \sqrt{14}$$

$$BC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(8-6)^2+(-2-2)^2+(-7+1)^2} = \sqrt{4+16+36} = \sqrt{56} = 2\sqrt{14}$$

$$AC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(8-5)^2+(-2-4)^2+(-7-2)^2} = \sqrt{9+36+81} = \sqrt{126} = 3\sqrt{14}$$

$\therefore\ AB+BC=AC\ , \therefore A, B, C$ *are collinear*

21. *If* $(3,\ 2,\ -1)\ (\ 4,\ 1,\ 1)$ *and* $(\ 6,\ 2,5)$ *are three vertices and* $(4,2,2)$ *is the centroid of a tetrahedron, find the fourth vertex*

sol. given $A(3,\ 2,\ -1)\ B(\ 4,\ 1,\ 1)\ C(\ 6,\ 2,5)$ *and* $D(x,y,z)$ *are vertices of tetrahedron*

given $G(4,2,2)$ *is the centroid of a tetrahedron*

$$we\ know\ that\ G=\left(\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}, \frac{z_1+z_2+z_3+z_4}{4}\right)$$

$$(4,2,2)=\left(\frac{3+4+6+x}{4}, \frac{2+1+2+y}{4}, \frac{-1+1+5+z}{4}\right) \Rightarrow 4=\frac{3+4+6+x}{4},\ 2=\frac{2+1+2+y}{4},\ 2=\frac{-1+1+5+z}{4}$$

$$\Rightarrow 16=13+x\ ,\ 8=5+y\ , 8=5+z \qquad \Rightarrow x=3, y=3, z=3$$

Centroid G $= \left(\frac{\quad}{4}, \frac{\quad}{4}, \frac{\quad}{4} \right)$

$$G = \left(\frac{2-3-1+3}{4}, \frac{3+3+4+5}{4}, \frac{-4-2+2+1}{4} \right) = \left(\frac{1}{4}, \frac{15}{4}, \frac{-3}{4} \right)$$

23. *Find the coordinates of the vertices 'C' of ΔABC, if its centroid is the origin and vertices are* $(1,1,1)$ *and* $(-2,4,1)$

Sol. Let $A=(1,1,1)$, $B=(-2,4,1)$, $C=(x,y,z)$ *and* $G=(0,0,0)$

we know that $G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3} \right)$

$$\Rightarrow (0,0,0) = \left(\frac{1-2+x}{3}, \frac{1+4+y}{3}, \frac{1+1+z}{3} \right)$$

$$\Rightarrow (0,0,0) = \left(\frac{x-1}{3}, \frac{y+5}{3}, \frac{z+2}{3} \right)$$

$$\Rightarrow \frac{x-1}{3} = 0, \frac{y+5}{3} = 0, \frac{z+2}{3} = 0$$

$$\Rightarrow x = 1, y = -5, z = -2$$

24. *Find the equation of the plane if the foot of the perpendicular from origin to the plane is* $(1,3,-5)$

sol. Given points $O(x_1,y_1,z_1)=(0,0,0)$ *and* $P(x_2,y_2,z_2)=(1,3,-5)$

The dr's of OP are $(a,b,c)=(1-0,3-0,-5-0)=(1,3,-5)$

$\therefore$ *the equation of the plane is* $a(x-x_2)+b(y-y_2)+c(z-z_2)=0$

$\Rightarrow 1(x-1)+3(y-3)-5(z+5)=0 \quad \Rightarrow x-1+3y-9-5z-25=0 \quad \Rightarrow x+3y-5z-35=0$

25. *i) Find the angle between the planes* $x+2y+2z-5=0$ *and* $3x+3y+2z-8=0$

ii) Find the angle between the planes $2x-y+z-6=0$ *and* $x+y+2z=7$

sol. the angle between the planes $a_1x+b_1y+c_1z+d_1=0$ *and* $a_2x+b_2y+c_2z+d_2=0$ *is*

$$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

given planes $x+2y+2z-5=0$ *and* $3x+3y+2z-8=0$

$$\cos\theta = \frac{1.3+2.3+2.2}{\sqrt{1^2+2^2+2^2}\sqrt{3^2+3^2+2^2}} \Rightarrow \cos\theta = \frac{3+6+4}{\sqrt{1+4+4}\sqrt{9+9+4}} \Rightarrow \cos\theta = \frac{13}{3\sqrt{22}}$$

26. *Find the equation of the plane passing through point* $(1,1,1)$ *and parallel to the plane* $x+2y+3z-7=0$

Sol. Given plane is $x+2y+3z-7=0 \Rightarrow a=1, b=2, c=3$

given point $(x_1,y_1,z_1)=(1,1,1)$

the required equation of the plane is $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$

$\Rightarrow 1(x-1)+2(y-1)+3(z-1)=0 \quad \Rightarrow x+2y+3z-6=0$

27. *Find the direction cosines of the normal to the plane* $x+2y+2z-4=0$

Sol. Given plane $x+2y+2z-4=0$ *ie,* $a=1, b=2, c=2$

divide the plane with $\sqrt{a^2+b^2+c^2} = \sqrt{1^2+2^2+2^2} = \sqrt{1+4+4} = 3$

ie, $\frac{x}{3}+\frac{2y}{3}+\frac{2z}{3}-\frac{4}{3}=0 \qquad \therefore$ *The direction cosines of normal plane is* $\left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3} \right)$

$$ie, \frac{x}{\sqrt{14}} + \frac{2y}{\sqrt{14}} - \frac{3z}{\sqrt{14}} - \frac{6}{\sqrt{14}} = 0$$

$$\therefore \text{The equation of plane in normal form is } \frac{x}{\sqrt{14}} + \frac{2y}{\sqrt{14}} - \frac{3z}{\sqrt{14}} = \frac{6}{\sqrt{14}}$$

29. *Write the equation of plane* $4x - 4y + 2z + 5 = 0$ *in the intercept form*

Sol. Given plane $4x - 4y + 2z + 5 = 0$

$$\Rightarrow 4x - 4y + 2z = -5 \qquad \Rightarrow \frac{4x - 4y + 2z}{-5} = 1$$

$$\Rightarrow \frac{4x}{-5} - \frac{4y}{-5} + \frac{2z}{-5} = 1 \qquad \Rightarrow \frac{x}{-5/4} + \frac{y}{-5/-4} + \frac{z}{-5/2} = 1$$

$$\therefore \; x-\text{intercept} = \frac{-5}{4} \;, y-\text{intercept} = \frac{5}{4} \;, z-\text{intercept} = \frac{-5}{2}$$

30. *Find the equation of the plane whose intercepts on X, Y, Z axis are* $1, 2, 4$ *respectively*

Sol. Given $x-\text{intercept}, a = 1 \;;\; y-\text{intercept}, b = 2 \;;\; z-\text{intercept}, c = 4$

the equation of plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \quad ie, \frac{x}{1} + \frac{y}{2} + \frac{z}{4} = 1$

31. *Compute* $\lim_{x\to 0}\left(\frac{a^x - 1}{b^x - 1}\right)(a > b > 0, b \neq 1)$

$$Sol. \lim_{x\to 0}\left(\frac{a^x - 1}{b^x - 1}\right) = \left(\frac{\lim_{x\to 0}\frac{a^x - 1}{x}}{\lim_{x\to 0}\frac{b^x - 1}{x}}\right) = \frac{\log_e a}{\log_e b} = \log_b a$$

32. *Find* $\lim_{x\to 0}\left(\frac{e^x - \sin x - 1}{x}\right)$

$$Sol. \lim_{x\to 0}\left(\frac{e^x - \sin x - 1}{x}\right)$$

$$= \lim_{x\to 0}\left(\frac{e^x - 1}{x}\right) - \lim_{x\to 0}\left(\frac{\sin x}{x}\right) = 1 - 1 = 0$$

33. *Compute* $\lim_{x\to 0}\frac{e^{7x} - 1}{x}$

$$Sol. \lim_{x\to 0}\frac{e^{7x} - 1}{x}$$

$$= \lim_{x\to 0}\frac{e^{7x} - 1}{7x} X7 = 1X7 = 7$$

34. *Find* $\lim_{x\to 0}\frac{\sqrt{1+x} - 1}{x}$

$$Sol. \lim_{x\to 0}\frac{\sqrt{1+x} - 1}{x}$$

$$= \lim_{x\to 0}\frac{\sqrt{1+x} - 1}{x} X \frac{\sqrt{1+x} + 1}{\sqrt{1+x} + 1}$$

$$= \lim_{x\to 0}\frac{1 + x - 1}{x\left(\sqrt{1+x} + 1\right)}$$

$$= \lim_{x\to 0}\frac{1}{\sqrt{1+x} + 1} = \frac{1}{\sqrt{1+0} + 1} = \frac{1}{2}$$

35. *Compute* $\lim_{x\to\infty}\frac{11x^3 - 3x + 4}{13x^3 - 5x^2 - 7}$

$$Sol. \lim_{x\to\infty}\frac{11x^3 - 3x + 4}{13x^3 - 5x^2 - 7}$$

$$= \lim_{x\to\infty}\frac{x^3\left[11 - 3\frac{1}{x^2} + 4\frac{1}{x^3}\right]}{x^3\left[13 - 5\frac{1}{x} - 7\frac{1}{x^3}\right]}$$

$$= \frac{11 - 0 + 0}{13 - 0 - 0} = \frac{11}{13}$$

36. *Compute* $\lim_{x\to\infty}\frac{x^2 + 5x + 2}{2x^2 - 5x + 1}$

$$Sol. \lim_{x\to\infty}\frac{x^2 + 5x + 2}{2x^2 - 5x + 1}$$

$$= \lim_{x\to\infty}\frac{x^2\left[1 + 5\frac{1}{x} + 2\frac{1}{x^2}\right]}{x^2\left[2 - 5\frac{1}{x} + 1\frac{1}{x^2}\right]}$$

$$= \frac{1 + 0 + 0}{2 - 0 + 0} = \frac{1}{2}$$

37. *Find* $\lim_{x\to 0^+}\left(\frac{2|x|}{x} + x + 1\right)$

Sol. As $x \to 0^+ \Rightarrow x > o \Rightarrow |x| = x$

now $\lim_{x\to 0^+}\left(\frac{2|x|}{x} + x + 1\right) = \lim_{x\to 0^+}\left(\frac{2x}{x} + x + 1\right) = 2 + 0 + 1 = 3$

38. *Compute* $\lim_{x\to 0}\frac{\tan(x-a)}{x^2 - a^2}(a \neq 0)$

$$Sol. \lim_{x\to 0}\frac{\tan(x-a)}{x^2 - a^2}$$

$$= \lim_{x\to 0}\frac{\tan(x-a)}{(x-a)(x+a)} = \lim_{x\to 0}\frac{\tan(x-a)}{(x-a)}.\frac{1}{(x+a)} = \frac{1}{a+a} = \frac{1}{2a}$$

$$Sol.\lim_{x\to 0}\left(\frac{e^x-1}{\sqrt{1+x}-1}\right)$$

$$=\lim_{x\to 0}\left(\frac{e^x-1}{\sqrt{1+x}-1}\cdot\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}\right)$$

$$=\lim_{x\to 0}\left(\frac{e^x-1}{1+x-1}\cdot\left(\sqrt{1+x}+1\right)\right)$$

$$=\lim_{x\to 0}\left(\frac{e^x-1}{x}\right)\cdot\left(\sqrt{1+x}+1\right)$$

$$=1.\left(\sqrt{1+0}+1\right)$$

$$=1+1=2$$

$$Sol.\lim_{x\to 0}\frac{x(e^x-1)}{1-\cos x}$$

$$=\lim_{x\to 0}\frac{(e^x-1)}{x}\,\frac{x^2}{1-\cos x}$$

$$=\lim_{x\to 0}\frac{(e^x-1)}{x}\ \mathrm{X}\lim_{x\to 0}\frac{x^2}{2\sin^2 x/2}$$

$$=\lim_{x\to 0}\frac{(e^x-1)}{x}\mathrm{X}\frac{1}{2}\mathrm{X}\frac{1}{\left(\lim_{x\to 0}\frac{\sin x/2}{x/2}\right)^2\mathrm{X}\left(\frac{1}{2}\right)^2}$$

$$=1\mathrm{X}\frac{1}{2}\mathrm{X}\frac{1}{1\mathrm{X}\frac{1}{4}}=2$$

$$\lim_{x\to 0}\left(\frac{\quad}{\sin^2 nx}\right)$$

$$=\lim_{x\to 0}\left(\frac{2\sin^2 mx}{\sin^2 nx}\right)$$

$$=2.\frac{\left(\lim_{x\to 0}\frac{\sin mx}{mx}\right)^2}{\left(\lim_{x\to 0}\frac{\sin nx}{nx}\right)^2}$$

$$=2.\frac{m^2}{n^2}$$

42. *Compute* $\lim_{x\to 3}\frac{x^2-8x+15}{x^2-9}$

$$Sol.\lim_{x\to 3}\frac{x^2-8x+15}{x^2-9}$$

$$=\lim_{x\to 3}\frac{x^2-5x-3x+15}{(x-3)(x+3)}$$

$$=\lim_{x\to 3}\frac{(x-3)(x-5)}{(x-3)(x+3)}$$

$$=\frac{3-5}{3+3}=\frac{-2}{6}=\frac{-1}{3}$$

43. *Compute* $\lim_{x\to 2}\left[\frac{1}{x-2}-\frac{4}{x^2-4}\right]$

$$Sol.\lim_{x\to 2}\left[\frac{1}{x-2}-\frac{4}{x^2-4}\right]$$

$$=\lim_{x\to 2}\left[\frac{x+2-4}{x^2-4}\right]$$

$$=\lim_{x\to 2}\left[\frac{x-2}{(x-2)(x+2)}\right]$$

$$=\lim_{x\to 2}\frac{1}{x+2}=\frac{1}{2+2}=\frac{1}{4}$$

44. *Compute* $\lim_{x\to 0}x^2\sin\left(\frac{1}{x}\right)$

Sol. we know that $-1\le\sin\left(\frac{1}{x}\right)\le 1$

$$\Rightarrow -x^2\le x^2\sin\left(\frac{1}{x}\right)\le x^2$$

$$\therefore \lim_{x\to 0}(-x^2)=0\ and\ \lim_{x\to 0}(x^2)=0$$

$$\therefore \lim_{x\to 0}x^2\sin\left(\frac{1}{x}\right)=0$$

45. *Find* $\lim_{x\to\infty}\left(\frac{8|x|+3x}{3|x|-2x}\right)$

Sol. As $x\to\infty\Rightarrow x>o\Rightarrow|x|=x$

$$now\lim_{x\to\infty}\left(\frac{8|x|+3x}{3|x|-2x}\right)$$

$$=\lim_{x\to\infty}\left(\frac{8x+3x}{3x-2x}\right)=\lim_{x\to\infty}\left(\frac{11x}{x}\right)=11$$

47. *Evaluate* $\lim_{x\to 1}\frac{\log_e x}{x-1}$

Sol. put $x-1=y\Rightarrow x=y+1$

as $x\to 1\Rightarrow y\to 0$

$$now\lim_{x\to 1}\frac{\log_e x}{x-1}=\lim_{y\to 0}\frac{\log_e(1+y)}{y}=1$$

48. *Compute* $\lim_{x\to 0}\frac{e^{3x}-1}{x}$

$$Sol.\lim_{x\to 0}\frac{e^{3x}-1}{x}$$

$$=\lim_{x\to 0}\frac{e^{3x}-1}{3x}\mathrm{X}3=1\mathrm{X}3=3$$

46. *Compute* $\lim_{x\to 2}([x]+x)$

Sol. As $x\to a^+\Rightarrow[x]=a$ *, As* $x\to a^-\Rightarrow[x]=a-1$

As $x\to 2^+\Rightarrow[x]=2$ *, As* $x\to 2^-\Rightarrow[x]=2-1=1$

$$\lim_{x\to 2^+}([x]+x)=2+2=4\qquad\lim_{x\to 2^-}([x]+x)=1+2=3$$

$$\therefore \lim_{x\to 2^+}([x]+x)\ne\lim_{x\to 2^-}([x]+x)$$

$\therefore \lim_{x\to 2}([x]+x)$ *does not exist*

51. *Is the function defined by* $f(x)=\begin{cases}\frac{\sin 2x}{x} & if\ x\ne 0\\ 1 & if\ x=0\end{cases}$ *is continuous at* $x=0$?

Sol. Given that $f(x)=\frac{\sin 2x}{x}$ *if* $x\ne 0$ *and* $f(0)=1$

$$now\lim_{x\to 0}f(x)=\lim_{x\to 0}\frac{\sin 2x}{x}=2.\lim_{x\to 0}\frac{\sin 2x}{2x}=2.1=2$$

$\therefore \lim_{x\to 0}f(x)\ne f(0)\ \therefore f$ *is not continuous at* $x=0$

$\Rightarrow 1 \ge -\sin x \ge -1$

$\Rightarrow -1 \le -\sin x \le 1$

$\Rightarrow x^2 - 1 \le x^2 - \sin x \le x^2 + 1$

$\Rightarrow \dfrac{x^2-1}{x^2-2} \le \dfrac{x^2-\sin x}{x^2-2} \le \dfrac{x^2+1}{x^2-2}$

$$\therefore \lim_{x\to\infty} \frac{x^2-1}{x^2-2} = \lim_{x\to\infty} \frac{x^2\left(1-\frac{1}{x^2}\right)}{x^2\left(1-\frac{2}{x^2}\right)} = \frac{1-0}{1-0} = 1$$

$$\therefore \lim_{x\to\infty} \frac{x^2+1}{x^2-2} = \lim_{x\to\infty} \frac{x^2\left(1+\frac{1}{x^2}\right)}{x^2\left(1-\frac{2}{x^2}\right)} = \frac{1+0}{1-0} = 1$$

$$\therefore \lim_{x\to\infty} \frac{x^2-\sin x}{x^2-2} = 1$$

$$= \lim_{x\to\infty} \left(\sqrt{x+1}-\sqrt{x}\right) \mathrm{X} \frac{\left(\sqrt{x+1}+\sqrt{x}\right)}{\left(\sqrt{x+1}+\sqrt{x}\right)}$$

$$= \lim_{x\to\infty} \frac{x+1-x}{\left(\sqrt{x+1}+\sqrt{x}\right)} = \lim_{x\to\infty} \frac{1}{\sqrt{x}\left(\sqrt{1+\frac{1}{\sqrt{x}}}+1\right)}$$

$$= \lim_{x\to\infty} \frac{1/\sqrt{x}}{\sqrt{1+\frac{1}{\sqrt{x}}}+1} = \frac{0}{1+1} = 0$$

52. *Is the function f, defined by* $f(x) = \begin{cases} x^2 & if\ x \le 1 \\ x & if\ x > 1 \end{cases}$*, continuous on R?*

Sol. Given $f(x) = x^2, if\ x < 1$

$f(x) = x^2, if\ x = 1$

$f(x) = x, if\ x > 1$

now $\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} x = \lim\limits_{h\to 0}(1+h) = 1+0 = 1$

also $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} x^2 = \lim\limits_{h\to 0}(1-h)^2 = (1-0)^2 = 1$

and $f(1) = 1^2 = 1$

$\therefore \lim\limits_{x\to 1^+} f(x) = f(1) = \lim\limits_{x\to 1^-} f(x)$

$\therefore$ f is continuous at $x = 1$

$\therefore$ f is continuous on R

53. *If* $y = \sin^{-1}\sqrt{x}$, *find* $\dfrac{dy}{dx}$

Sol. Given $y = \sin^{-1}\sqrt{x}$

diff. wrt x

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sin^{-1}\sqrt{x}\right]$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\sqrt{x}\right)^2}} \frac{d}{dx}\left(\sqrt{x}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}$$

54. *If* $y = e^{2x}.\log(3x+4)$ *find* $\dfrac{dy}{dx}$

Sol. Given $y = e^{2x}.\log(3x+4)$

$$\frac{dy}{dx} = \frac{d}{dx}\left[e^{2x}.\log(3x+4)\right]$$

$$\frac{dy}{dx} = e^{2x} \cdot \frac{d}{dx}\left[\log(3x+4)\right] + \left[\log(3x+4)\right] \cdot \frac{d}{dx} e^{2x}$$

$$\frac{dy}{dx} = e^{2x} \cdot \frac{1}{3x+4} \cdot \frac{d}{dx}(3x+4) + \left[\log(3x+4)\right].e^{2x} \cdot \frac{d}{dx} 2x$$

$$\frac{dy}{dx} = \frac{3e^{2x}}{3x+4} + 2e^{2x}[\log(3x+4)]$$

55. *Find the derivative of* $\cos^{-1}(4x^3 - 3x)$

Sol. let $y = \cos^{-1}(4x^3 - 3x)$

put $x = \cos\theta \Rightarrow \theta = \cos^{-1} x$

$\therefore\ y = \cos^{-1}(4\cos^2\theta - 3\cos\theta)$

$y = \cos^{-1}(\cos 3\theta)$

$y = 3\theta = 3\cos^{-1} x$

$$\frac{dy}{dx} = 3\frac{d}{dx}\cos^{-1} x$$

$$\frac{dy}{dx} = 3\left(\frac{-1}{\sqrt{1-x^2}}\right) = \left(\frac{-3}{\sqrt{1-x^2}}\right)$$

$Sol.Given\ y=\sec^{-1}\left(\frac{1}{2x^2-1}\right)$

$put\ x=\cos\theta\Rightarrow\theta=\cos^{-1}x$

$$\therefore y=\sec^{-1}\left(\frac{1}{2\cos^2\theta-1}\right)$$

$$y=\sec^{-1}\left(\frac{1}{\cos 2\theta}\right)=\sec^{-1}(\sec 2\theta)$$

$$y=2\theta=2\cos^{-1}x$$

$$\frac{dy}{dx}=2\frac{d}{dx}\cos^{-1}x$$

$$\frac{dy}{dx}=2\left(\frac{-1}{\sqrt{1-x^2}}\right)=\frac{-2}{\sqrt{1-x^2}}$$

57. *If* $y=\left(\cot^{-1}x^3\right)^2$, *find* $\frac{dy}{dx}$

$Sol.Given\ y=\left(\cot^{-1}x^3\right)^2$

$$\frac{dy}{dx}=\frac{d}{dx}\left(\cot^{-1}x^3\right)^2$$

$$\frac{dy}{dx}=2\left(\cot^{-1}x^3\right)\frac{d}{dx}\left(\cot^{-1}x^3\right)$$

$$\frac{dy}{dx}=2\left(\cot^{-1}x^3\right).\frac{-1}{1+\left(x^3\right)^2}.\frac{d}{dx}\left(x^3\right)$$

$$\frac{dy}{dx}=2\left(\cot^{-1}x^3\right).\frac{-1}{1+\left(x^3\right)^2}.3x^2$$

$$\frac{dy}{dx}=\frac{-6x^2(\cot^{-1}x^3)}{1+x^6}$$

58. *If* $y=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$, *find* $\frac{dy}{dx}$

$Sol.Given\ y=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$put\ x=\tan\theta\Rightarrow\theta=\tan^{-1}x$

$$\therefore\ y=\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$y=\tan^{-1}(\tan 2\theta)$$

$$y=2\theta=2\tan^{-1}x$$

$$\frac{dy}{dx}=2\frac{d}{dx}\tan^{-1}x$$

$$\frac{dy}{dx}=2\left(\frac{1}{1+x^2}\right)=\left(\frac{2}{1+x^2}\right)$$

59. *If* $f(x)=\log(\sec x+\tan x)$, *then find* $f^1(x)$

$Sol.Given\ f(x)=\log(\sec x+\tan x)$

$$f^1(x)=\frac{df}{dx}=\frac{d}{dx}\left(\log(\sec x+\tan x)\right)$$

$$f^1(x)=\frac{1}{\sec x+\tan x}\frac{d}{dx}\left(\sec x+\tan x\right)$$

$$f^1(x)=\frac{1}{\sec x+\tan x}\left(\sec x.\tan x+\sec^2 x\right)$$

$$f^1(x)=\frac{1}{\sec x+\tan x}\left(\left(\sec x\right)(\tan x+\sec x)\right)$$

$$f^1(x)=\sec x$$

60. *If* $f(x)=1+x+x^2+---+x^{100}$, *find* $f^1(x)$

$Sol.Given\ f(x)=1+x+x^2+---+x^{100}$

$$f^1(x)=0+1+2x+---+100x^{99}$$

$$f^1(1)=0+1+2+3+---+100$$

$$f^1(1)=\sum 100=\frac{100(100+1)}{2}=\frac{100(101)}{2}=5050$$

61. *If* $y=ae^{nx}+be^{-nx}$ *then prove that* $y''=n^2y$

$Sol.Given\ y=ae^{nx}+be^{-nx}$

$$\frac{dy}{dx}=a\frac{d}{dx}e^{nx}+b\frac{d}{dx}e^{-nx}$$

$$\frac{dy}{dx}=ae^{nx}.n+be^{-nx}.(-n)$$

$$\frac{d^2y}{dx^2}=an\frac{d}{dx}e^{nx}-bn\frac{d}{dx}e^{-nx}$$

$$\frac{d^2y}{dx^2}=an.e^{nx}.n-bne^{-nx}.(-n)$$

$$\frac{d^2y}{dx^2}=n^2\left[ae^{nx}+be^{-nx}\right]$$

$$\frac{d^2y}{dx^2}=n^2y$$

$Sol.Given\ y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$put\ x=\tan\theta \Rightarrow \theta=\tan^{-1}x$

$\therefore\ y=\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$y=\sin^{-1}(\sin 2\theta)$

$y=2\theta=2\tan^{-1}x$

$\frac{dy}{dx}=2\frac{d}{dx}\tan^{-1}x$

$\frac{dy}{dx}=2\left(\frac{1}{1+x^2}\right)=\left(\frac{2}{1+x^2}\right)$

63. $If\ \ y=\frac{2x+3}{4x+5},\ find\ \frac{dy}{dx}$

$Sol.Given\ y=\frac{2x+3}{4x+5}$

$\frac{dy}{dx}=\frac{d}{dx}\left[\frac{2x+3}{4x+5}\right]$

$\frac{dy}{dx}=\frac{(4x+5)\frac{d}{dx}(2x+3)-(2x+3)\frac{d}{dx}(4x+5)}{(4x+5)^2}$

$\frac{dy}{dx}=\frac{(4x+5).2-(2x+3).4}{(4x+5)^2}=\frac{-2}{(4x+5)^2}$

64. $If\ \ y=\tan^{-1}(\log x),\ find\ \frac{dy}{dx}$

$Sol.Given\ y=\tan^{-1}(\log x)$

$\frac{dy}{dx}=\frac{d}{dx}\left[\tan^{-1}(\log x)\right]$

$\frac{dy}{dx}=\frac{1}{1+(\log x)^2}.\frac{d}{dx}(\log x)$

$\frac{dy}{dx}=\frac{1}{1+(\log x)^2}.\frac{1}{x}$

65. $Find\ \ the\ derivative\ of\ \ f(x)=(x^2-3)(4x^3+1)$

$Sol.Given\ f(x)=(x^2-3)(4x^3+1)$

$f^1(x)=\frac{d}{dx}\left[(x^2-3)(4x^3+1)\right]$

$f^1(x)=(x^2-3)\frac{d}{dx}(4x^3+1)+(4x^3+1)\frac{d}{dx}(x^2-3)$

$f^1(x)=(x^2-3)(4.3x^2+0)+(4x^3+1)(2x-0)$

$f^1(x)=12x^2(x^2-3)+2x(4x^3+1)$

$f^1(x)=20x^4-36x^2+2x$

66. $If\ \ y=\sqrt{2x-3}+\sqrt{7-3x},\ find\ \frac{dy}{dx}$

$Sol.Given\ y=\sqrt{2x-3}+\sqrt{7-3x}$

$\frac{dy}{dx}=\frac{d}{dx}\sqrt{2x-3}+\frac{d}{dx}\sqrt{7-3x}$

$\frac{dy}{dx}=\frac{1}{2\sqrt{2x-3}}.\frac{d}{dx}(2x-3)+\frac{1}{2\sqrt{7-3x}}.\frac{d}{dx}(7-3x)$

$\frac{dy}{dx}=\frac{1}{2\sqrt{2x-3}}.(2)+\frac{1}{2\sqrt{7-3x}}.(0-3)$

$\frac{dy}{dx}=\frac{1}{\sqrt{2x-3}}-\frac{3}{2\sqrt{7-3x}}$

67. $If\ \ f(x)=2x^2+3x-5\ then\ PT\ \ f(0)+3f(-1)=0$

$Sol.Given\ f(x)=2x^2+3x-5 \Rightarrow f^1(x)=4x+3$

$f^1(0)=4(0)+3=3\ ,\ f^1(-1)=4(-1)+3=-1$

$\therefore f(0)+3f(-1)=3+3(-1)=3-3=0$

$\therefore\ f(0)+3f(-1)=0$

68. $If\ \ x=a\cos^3 t,\ y=a\sin^3 t,\ find\ \frac{dy}{dx}$

$Sol.Given\ x=a\cos^3 t$

$\frac{dx}{dt}=a\frac{d}{dt}\left(\cos^3 t\right)$

$\frac{dx}{dt}=a.3\cos^2 t\frac{d}{dt}\cos t$

$\frac{dx}{dt}=3a\cos^2 t(-\sin t)$

$\frac{dx}{dt}=-3a\cos^2 t\sin t$

$also\ \ y=a\sin^3 t$

$\frac{dy}{dt}=a\frac{d}{dt}\left(\sin^3 t\right)$

$\frac{dy}{dt}=a.3\sin^2 t\frac{d}{dt}\sin t$

$\frac{dx}{dt}=3a\sin^2 t(\cos t)$

$\frac{dx}{dt}=3a\cos t\sin^2 t$

$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{3a\cos t\sin^2 t}{-3a\cos^2 t\sin t}=-\tan t$

Sol.Given $y=\sec(\sqrt{\tan x})$

$$\frac{dy}{dx}=\frac{d}{dx}\left[\sec(\sqrt{\tan x})\right]$$

$$\frac{dy}{dx}=\sec(\sqrt{\tan x})\tan(\sqrt{\tan x})\frac{d}{dx}(\sqrt{\tan x})$$

$$\frac{dy}{dx}=\sec(\sqrt{\tan x})\tan(\sqrt{\tan x})\frac{1}{2\sqrt{\tan x}}\frac{d}{dx}(\tan x)$$

$$\frac{dy}{dx}=\sec(\sqrt{\tan x})\tan(\sqrt{\tan x})\frac{\sec^2 x}{2\sqrt{\tan x}}$$

Sol.Given $y=\sin^{-1}(\cos x)$

$$\frac{dy}{dx}=\frac{d}{dx}\left[\sin^{-1}(\cos x)\right]$$

$$\frac{dy}{dx}=\frac{1}{\sqrt{1-\cos^2 x}}\frac{d}{dx}\cos x$$

$$\frac{dy}{dx}=\frac{1}{\sqrt{\sin^2 x}}(-\sin x)$$

$$\frac{dy}{dx}=-1$$

71. *If* $y=5\sin x+e^x\log x$, *find* $\frac{dy}{dx}$

Sol. Given $y=5\sin x+e^x\log x$

$$\frac{dy}{dx}=5\frac{d}{dx}\sin x+\frac{d}{dx}\left[e^x\log x\right]$$

$$\frac{dy}{dx}=5\cos x+e^x\frac{d}{dx}\log x+\log x\frac{d}{dx}e^x$$

$$\frac{dy}{dx}=5\cos x+\frac{e^x}{x}+e^x\log x$$

72. *If* $f\left(x\right)=7^{x^3+3x}\left(x>0\right)$ *find* $f^{'}(x)$

sol.Given $y=7^{x^3+3x}$

apply log *on both side*

$$\Rightarrow \log y=\log\left(7^{x^3+3x}\right)$$

$$\Rightarrow \log y=\left(x^3+3x\right).\log 7$$

$$\Rightarrow \frac{d}{dx}\left(\log y\right)=\log 7.\frac{d}{dx}\left(x^3+3x\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\log 7.\left(3x^2+3\right)$$

$$\Rightarrow \frac{dy}{dx}=y\log 7.\left(3x^2+3\right)$$

$$\Rightarrow \frac{dy}{dx}=7^{x^3+3x}.\log 7.\left(3x^2+3\right)$$

73. *If* $y=\sin^{-1}(3x-4x^3)$, *find* $\frac{dy}{dx}$

Sol.Given $y=\sin^{-1}(3x-4x^3)$

put $x=\sin\theta\Rightarrow\theta=\sin^{-1}x$

$$\therefore y=\sin^{-1}(3\sin\theta-4\sin^3\theta)$$

$$y=\sin^{-1}(\sin 3\theta)$$

$$y=3\theta=3\sin^{-1}x$$

$$\frac{dy}{dx}=3\frac{d}{dx}\left(\sin^{-1}x\right)$$

$$\frac{dy}{dx}=3\frac{1}{\sqrt{1-x^2}}=\frac{3}{\sqrt{1-x^2}}$$

74. *If* $2x^2-3xy+y^2+x+2y-8=0$, *then find* $\frac{dy}{dx}$

Sol.Given $2x^2-3xy+y^2+x+2y-8=0$

diff .wrt x

$$\Rightarrow 2\frac{d}{dx}x^2-3\left(\frac{d}{dx}xy\right)+\frac{d}{dx}y^2+\frac{d}{dx}x+2\frac{d}{dx}y-\frac{d}{dx}8=0$$

$$\Rightarrow 2(2x)-3\left(x\frac{d}{dx}y+y\frac{d}{dx}x\right)+2y\frac{d}{dx}y+1+2\frac{dy}{dx}-0=0$$

$$\Rightarrow 4x-3x\frac{dy}{dx}-3y+2y\frac{dy}{dx}+1+2\frac{dy}{dx}=0$$

$$\Rightarrow (4x-3y+1)-\frac{dy}{dx}(3x-2y-2)=0$$

$$\Rightarrow \frac{dy}{dx}=\frac{4x-3y+1}{3x-2y-2}$$

$Sol.Given\ y = x + \tan x$

$\frac{dy}{dx} = \frac{d}{dx}x + \frac{d}{dx}\tan x$

$\frac{dy}{dx} = 1 + \sec^2 x$

$\frac{d^2y}{dx^2} = \frac{d}{dx}1 + \frac{d}{dx}\sec^2 x$

$\frac{d^2y}{dx^2} = 0 + 2\sec x.\frac{d}{dx}\sec x$

$\frac{d^2y}{dx^2} = 2\sec x.\sec x.\tan x$

$\frac{d^2y}{dx^2} = 2\sec^2 x.\tan x$

$\frac{d^2y}{dx^2} = \frac{2\tan x}{\cos^2 x}$

$\cos^2 x\frac{d^2y}{dx^2} = 2\tan x$

$\cos^2 x\frac{d^2y}{dx^2} = 2(y - x)$

$\cos^2 x\frac{d^2y}{dx^2} = 2y - 2x$

$\cos^2 x\frac{d^2y}{dx^2} + 2x = 2y$

78. *If* $y = ax^{n+1} + bx^{-n}$ *then prove that* $x^2y'' = n(n+1)y$

$Sol.Given\ y = ax^{n+1} + bx^{-n}$

$\frac{dy}{dx} = a\frac{d}{dx}x^{n+1} + b\frac{d}{dx}x^{-n}$

$\frac{dy}{dx} = ax^n.(n+1) + bx^{-n-1}.(-n)$

$\frac{d^2y}{dx^2} = a(n+1)\frac{d}{dx}x^n - bn\frac{d}{dx}x^{-n-1}$

$\frac{d^2y}{dx^2} = a(n+1)n.x^{n-1} - bnx^{-n-2}.(-n-1)$

$x^2\frac{d^2y}{dx^2} = n(n+1)\left[ax^{n+1} + bx^{-n}\right]$

$x^2\frac{d^2y}{dx^2} = n(n+1)y$

$\Rightarrow \operatorname{cosec} y = e^{2x+1}$ --------(1)

$\Rightarrow \frac{d}{dx}(\operatorname{cosec} y) = \frac{d}{dx}(e^{2x+1})$

$\Rightarrow -\operatorname{cosec} y.\cot y.\frac{dy}{dx} = e^{2x+1}.\frac{d}{dx}(2x+1)$

$\Rightarrow -\operatorname{cosec} y.\cot y.\frac{dy}{dx} = \operatorname{cosec} y.2$

$\Rightarrow \frac{dy}{dx} = \frac{-2}{\cot y} = -2\tan y$

77. *If* $x = \tan(e^{-y}), ST\ \frac{dy}{dx} = \frac{e^{-y}}{1+x^2}$

$Sol.Given\ x = \tan(e^{-y})$

$\tan^{-1} x = e^{-y}$

$\frac{d}{dx}\tan^{-1} x = \frac{d}{dx}e^{-y}$

$\frac{1}{1+x^2} = e^{-y}\frac{d}{dx}(-y)$

$\frac{1}{1+x^2} = -e^{-y}\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{-1}{e^{-y}(1+x^2)}$

$\frac{dy}{dx} = \frac{-e^y}{1+x^2}$

79. *Find the approximate value of* $\sqrt{82}$

$Sol.Let\ f(x) = \sqrt{x} \Rightarrow f'(x) = \frac{1}{2\sqrt{x}}$

$put\ x = 81\ \ and\ \ \Delta x = 1$

$now\ f(x+\Delta x) = f(x) + f'(x)\Delta x$

$\Rightarrow f(81+1) = \sqrt{81} + \frac{1}{2\sqrt{81}}.1$

$\Rightarrow f(82) = 9 + \frac{1}{18}$

$\Rightarrow \sqrt{82} = 9 + 0.055 = 9.0555$

increase in the area of square?

Sol. Let x be the side and A be area of square

$given \frac{dx}{x} X100 = 4$

$we\ know\ that\ A = x^2$

$\frac{dA}{dx} = 2x$

$dA = 2x\,dx$

$\frac{dA}{A} = \frac{2xdx}{x^2}$

$\frac{dA}{A} X100 = 2.\frac{dx}{x} X100 = 2.4 = 8$

$Sol. Given\ y = f(x) = x^2 + 3x + 6, x = 10, \Delta x = 0.01$

$f'(x) = \frac{d}{dx}(x^2 + 3x + 6) = 2x + 3$

$\Delta y = f(x + \Delta x) - f(x)$

$= f(10 + 0.01) - f(10)$

$= f(10.01) - f(10)$

$= \left[(10.01)^2 + 3(10.01) + 6\right] - \left[10^2 + 3(10) + 6\right]$

$= 100.2001 + 30.03 + 6 - 100 - 30 - 6$

$= 0.2301$

$dy = f'(x)\Delta x$

$dy = (2x + 3)0.01$

$dy = (23)0.01 = 0.23$

82. $If\ y = x^2 + x, x = 10, \Delta x = 0.1, then\ find\ \Delta y\ and\ dy?$

$Sol. Given\ y = f(x) = x^2 + x, x = 10, \Delta x = 0.1$

$f'(x) = \frac{d}{dx}(x^2 + x) = 2x + 1$

$\Delta y = f(x + \Delta x) - f(x)$

$= f(10 + 0.1) - f(10)$

$= f(10.1) - f(10)$

$= \left[(10.1)^2 + (10.1)\right] - \left[10^2 + (10)\right]$

$= 102.01 + 10.1 - 100 - 10$

$= 2.11$

$dy = f'(x)\Delta x$

$dy = (2x + 1)0.1$

$dy = (21)0.1 = 2.10$

83. $If\ y = e^x + x, x = 5, \Delta x = 0.02, then\ find\ \Delta y\ and\ dy?$

$Sol. Given\ y = f(x) = e^x + x, x = 5, \Delta x = 0.02$

$f'(x) = \frac{d}{dx}(e^x + x) = e^x + 1$

$\Delta y = f(x + \Delta x) - f(x)$

$= f(5 + 0.02) - f(5)$

$= f(5.02) - f(5)$

$= e^{5.02} + 5.02 - e^5 - 5$

$= e^{5.02} - e^5 + 0.02$

$dy = f'(x)\Delta x$

$dy = (e^x + 1)0.02$

$dy = (e^5 + 1)0.02$

84. *Find the approximate value of* $\sqrt[4]{17}$

$Sol. Let\ f(x) = \sqrt[4]{x} \Rightarrow f'(x) = \frac{1}{4} x^{-3/4}$

$put\ x = 16\ \ and\ \ \Delta x = 1$

$now\ f(x + \Delta x) = f(x) + f'(x)\Delta x$

$\Rightarrow f(16 + 1) = \sqrt[4]{16} + \frac{1}{4}.(16)^{-3/4}$

$\Rightarrow f(17) = 2 + \frac{1}{4}.\frac{1}{8}$

$\Rightarrow \sqrt[4]{17} = 2 + \frac{1}{32} = 2 + 0.03125 = 2.0312$

85. $If\ 0 \le x \le \frac{\pi}{2}, then\ prove\ that\ x \ge \sin x$

$Sol. Let\ f(x) = x - \sin x$

$f'(x) = 1 - \cos x \ge 0\ \ \forall x$

$\therefore f\ is\ increasing\ function$

$\therefore f(x) \ge f(0)$

$\Rightarrow x - \sin x \ge 0$

$\Rightarrow x \ge \sin x$

86. Define Rolle's theorem

Sol. if $f:[a,b] \to R$ *is a function such that i)* f *is continuous on* $[a,b]$

ii) f *is derivable on* (a,b) *and iii)* $f(a) = f(b)$ *then* $\exists c \in (a,b)$ *such that* $f'(c) = 0$

$Sol.Given \quad f(x)=x^2-5x+6 \Rightarrow f'(x)=2x-5$

$\therefore f$ *is continuous on* $[-3,8]$ *and derivable on* $(-3,8)$

also $f(-3)=(-3)^2-5(-3)+6=9+15+6=30$

$f(8)=8^2-5(8)+6=64-40+6=30$

$\therefore f(-3)=f(8)$

$\therefore f$ *satisfies all the conditions of Rolle's theorem*

$\therefore$ *there exists* $C\in(-3,8)$ *such that* $f'(C)=0$

$\Rightarrow 2c-5=0 \Rightarrow c=\dfrac{5}{2}\in(-3,8)$

Hence Rolle's theorem verified

89. *Verify Rolle's theorem for the function*

$f:[-1,1]\to R$ *be defined by* $f(x)=x^2-1$

$Sol.Given \quad f(x)=x^2-1 \Rightarrow f'(x)=2x$

$\therefore f$ *is continuous on* $[-1,1]$ *and derivable on* $(-1,1)$

also $f(-1)=(-1)^2-1=1-1=0$

$f(1)=1^2-1=1-1=0$

$\therefore f(-1)=f(1)$

$\therefore f$ *satisfies all the conditions of Rolle's theorem*

$\therefore$ *there exists* $C\in(-1,1)$ *such that* $f'(C)=0$

$\Rightarrow 2c=0 \Rightarrow c=0\in(-1,1)$

Hence Rolle's theorem verified

90. *Let* $f(x)=(x-1)(x-2)(x-3)$. *prove that there is more than one 'c' in* $(1,3)$ *such that* $f'(c)=0$

$Sol.Given \quad f(x)=(x-1)(x-2)(x-3)$

$\Rightarrow f(x)=x^3-6x^2+11x-6$

$\Rightarrow f'(x)=3x^2-12x+11$

$\therefore f$ *is continuous on* $[1,3]$ *and derivable on* $(1,3)$

$f(1)=0=f(3)$

$\therefore f$ *satisfies all the conditions of Rolle's theorem*

$\therefore$ *there exists* $C\in(1,3)$ *such that* $f'(c)=0$

$\Rightarrow 3c^2-12c+11=0$

$\Rightarrow C=\dfrac{12\pm\sqrt{144-132}}{6}$

$\Rightarrow C=2\pm\dfrac{1}{\sqrt{3}}$

$Sol.Given \quad f(x)=x^2+4 \Rightarrow f'(x)=2x$

$\therefore f$ *is continuous on* $[-3,3]$ *and derivable on* $(-3,3)$

also $f(-3)=(-3)^2+4=9+4=13$

$f(3)=3^2+4=9+4=13$

$\therefore f(-3)=f(3)$

$\therefore f$ *satisfies all the conditions of Rolle's theorem*

$\therefore$ *there exists* $C\in(-3,3)$ *such that* $f'(C)=0$

$\Rightarrow 2c=0 \Rightarrow c=0\in(-3,3)$

Hence Rolle's theorem verified

91. *Verify Rolle's theorem for the function*

$f:[-1,1]\to R$ *be defined by* $f(x)=\log(x^2+2)-\log 3$

$Sol.Given \quad f(x)=\log(x^2+2)-\log 3$

$\Rightarrow f'(x)=\dfrac{d}{dx}\left[\log(x^2+2)-\log 3\right]$

$\Rightarrow f'(x)=\dfrac{1}{x^2+2}\dfrac{d}{dx}(x^2+2)$

$\Rightarrow f'(x)=\dfrac{2x}{x^2+2}$

$\therefore f$ *is continuous on* $[-1,1]$ *and derivable on* $(-1,1)$

$\therefore f(-1)=f(1)=0$

$\therefore f$ *satisfies all the conditions of Rolle's theorem*

$\therefore$ *there exists* $C\in(-1,1)$ *such that* $f'(C)=0$

$\Rightarrow \dfrac{2c}{c^2+2}=0 \Rightarrow 2c=0 \Rightarrow c=0\in(-1,1)$

Hence Rolle's theorem verified

92. *Verify the conditions of the Lagrange's mean value theorem for* x^2-1 *on* $[2,3]$

$Sol.Given \quad f(x)=x^2-1 \Rightarrow f'(x)=2x$

$\therefore f$ *is continuous on* $[2,3]$ *and derivable on* $(2,3)$

also $f(2)=(2)^2-1=4-1=3$

$f(3)=3^2-1=9-1=8$

$\therefore f$ *satisfies all the conditions of Lagrange's theorem*

$\therefore$ *there exists* $C\in(-1,1)$ *such that*

$f'(C)=\dfrac{f(3)-f(2)}{3-2}=\dfrac{8-3}{1}=5$

$\Rightarrow 2c=5 \Rightarrow c=\dfrac{5}{2}\in(2,3)$

Hence Lagrange's theorem verified